

# Priority-Slot-Based Continuous-Time MINLP Formulation for Crude-Oil Scheduling Problem with Two-Pipeline Transportation

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**Abstract:** Some oil refineries arrange low-fusion-point crude oil storage tanks and ones containing high-fusion-point crude oil at two diverse locations. Refinery needs different pipeline to transport different fusion point oil, so, to transport diverse fusion point oil it needs two pipelines. Because of the constraints resulted from transporting high-fusion-point oil, to schedule such a system is very hard. Based on single-operation sequencing (SOS), this paper proposes a new MINLP formulation with transporting high-fusion-point oil and two pipelines transportation. The formulation is different from previous ones as it takes transporting high-fusion-point oil and two-pipeline transportation into account. It aims at minimizing the sum of high-fusion-point oil transportation cost. A simple two-stage MILP-NLP procedure is applied to solve this model and get a satisfactory optimality gap. Finally, this paper gives an intricate industrial example to illustrate application of the formulation.

**Keywords:** Oil refinery; Continuous-time formulation; Two-pipeline transportation, High-fusion-point oil transportation

## INTRODUCTION

A great challenge may occur during operating an oil refinery [Wu *et al.*, 2005 and 2009]. Generally, operating a plant has three hierarchies: planning production, scheduling production, as well controlling process. As is known that when well operating an oil refinery can augment benefit by \$10 per ton of product or more [Moro, 2003]. Therefore, as a typical process industry, many researchers have paid great attention to the effective technique development for the refinery operations. So far, in most oil refineries, sophisticated control units have been widely fixed for unit control at the controlling process hierarchy for optimizing partial production objectives, leading to prominent productivity gains in plant units.

Oil refineries are increasingly focusing on more desirable operation planning at planning production hierarchy. With using commercial software based on linear programming for refinery planning production, common production plan of whole refinery can be discovered. Pelham and Pharris pointed out in [1996] that planning technique can be reckoned advanced; as well related breakthrough should not be anticipated. The significant improvements in the field will be based on modeling method improvement by using nonlinear programming.

The middle hierarchy is short-term scheduling. As far as its modeling and solution algorithms are concerned, it is among hardest optimization problems. As was pointed out by Shobrys and White [2000], during efficiently operating a refinery, the three hierarchies should cooperate with each other. Thus,

based on the advanced techniques for planning and process controlling, it is critical to improve efficient short-term scheduling techniques [Gabbar, 2007]. However, without an efficient technique for optimal scheduling, it is impossible to get the global financial optima for a refinery.

Due to the NP-hard characteristic of general scheduling problems [Baker, 1996], commonly heuristics and meta-heuristics are employed to elucidate a scheduling optimization problem in discrete manufacturing operations [Chen *et al.*, 1998; Mattfeld and Bierwirth, 2004; Ponnambalam *et al.*, 1999; Sabuncuoglu and Bayiz, 1999; and Yang and Wang, 2001]. For the scheduling of a process plant, effort has been made by using rule-based algorithms [Stephanopoulos and Han, 1996], search algorithms [Murakami *et al.*, 1997], and petri net-based algorithms [Wu *et al.*, 2007, 2008a, 2008b, 2009, 2010a, 2010b, and 2011]. These techniques may not be capable of finding an optimal solution.

In solving scheduling problems, Constraint Programming (CP) is known to be very efficient [Baptiste, 2001]. However, it is seldom applied to solve problems appearing in the chemical engineering field. One of the reasons is that CP is effective to complete sequence of tasks or jobs which are well-defined in advance. Consequently, mixed integer programming-based techniques, especially mixed integer linear programming (MILP), have been preferred [Ierapetritou and Floudas, 1998a and 1998b; Mendez and Cerda, 2003; Pinto and Grossmann, 1997; Moro and Pinto, 2004; and Kallrath, 2002]. Based on time grids, a mixed integer programming formulation can easily model the capacity of a tank or

production unit at each time interval end [Floudas and Lin, 2004; and Mendez *et al.*, 2006].

Through mixed integer programming models, the uniform time discretization formulations have been successfully applied in solving batch processes representation based on an STN or RTN [Kondili *et al.*, 1993; and Pantelides *et al.*, 1997]. The formulation has duplex benefits: easy utilization in diverse problems and there being no nonlinear constraint in the model. However, while a huge time interval number is demanded in order to obtain an acceptable accuracy, the problem size becomes refractory even for efficient commercial solvers due to a huge binary variable number.

For the sake of reducing the discrete variable number of, non-uniform time discretization formulations have been set up on the basis of an RTN or STN representation [Zhang and Sargent, 1996; Schilling and Pantelides, 1996; and Lee *et al.*, 1996]. It differs from the discrete-time formulation mainly on the time slot duration that does not keep constant and has to be decided by a solver. This method is also to fulfill easily and is applicable for scheduling problems with a long horizon as it results in more abbreviated model. Despite having fewer binary variables, however, it has nonlinear constraints, and its linear programming (LP) relaxation is less tight generally, which causes the problem harder to elucidate.

In many refineries, the storage tanks are located at one site. First, refinery unloads crude oil of all types into storage tanks, and then delivers them to the charging ones through only one pipeline. In this situation, schedulability constraints are obtained on the hypothesis that there is no processing of high fusion point oil, whose abbreviation is H-oil [Wu, Zhou, and Chu 2008b; Wu *et al.* 2009, 2011]. When H-oil needs processing, the H-oil in storage tanks has to be delivered into charging tanks and setup cost of these delivery is very high, that farther makes the short-term scheduling problem complicated. Therefore, to constitute the schedulability constraints and determine how much crude oil can be transferred by an individual setup for the sake of minimizing the transfer cost becomes indispensable.

In many other situations, because of the particular prerequisite of H-oil offloading from a vessel, the refinery locates the storage tanks at two geographically diverse locations rather than one. The storage tanks at diverse locations are employed to store low fusion point crude oil (named L-oil for short) and H-oil, respectively. Because of two locations of storage tanks, crude oil must be delivered from storage tanks to charging tanks using two pipelines. Before transferring H-oil, the refinery must heat a certain pipeline first. When hot L-oil in the charging tanks flowing through the pipeline into storage ones, it becomes hot enough so that H-oil is capable of flowing in it. After finishing the H-oil transfer, it becomes at leisure. Nonetheless H-oil cannot sojourn and keep immobile, because

otherwise H-oil could be solidified so as to congest it. Therefore, one must exploit L-oil to run across it in the same orientation to eject H-oil totally. Evidently, the crude oil stream in it has bidirectional orientation. Thus, some charging tanks should not be employed in charging distillers but in transferring oil, which requires more charging tanks. In order to decrease operation cost, to transfer H-oil of a type from storage tanks as much as possible at a time to the refinery with an individual setup. As far as we've known, there is no research report to address such a refinery short-term scheduling using mathematical programming with the above-described pipelines.

Recently, Mouret *et al.* [2009] propose a continuous time formulation based on priority-slot for the crude oil operation scheduling problem. The most special advantage of such a formulation is that the total operation number to be performed by the obtained schedule is the only parameter that needs to be known in advance. However, in [Mouret *et al.*, 2009], to make the problem solvable, they do not consider oil residency time constraints that are solid and cannot be ignored in a real-life refinery and pipeline transfer, not to speak of H-oil and two pipelines transfer. Hence, by using the model in [Mouret *et al.*, 2009], an infeasible solution may be obtained. Notice that tackling L-oil and H-oil with two pipelines results in different operations, which makes the scheduling problem much more challenging. This work aims at creating a formulation based on single-operation sequencing (SOS) which can decide the required H-oil amount in storage tanks that can be delivered to charging ones, as well the way to transfer it. Hence, the oil delivery process makes the scheduling problem much more complicated. To obtain a feasible schedule, both H-oil transfer and oil delivery processes via two pipelines should be taken into account. This motivates us to conduct the study on the crude oil operation scheduling problem with H-oil transfer and oil delivery processes via two pipelines. The problem is formulated as a mixed integer non-linear programming with continuous-time representation by using the priority-slot-based modeling method.

This paper proposes a model using SOS modeling method [Mouret *et al.*, 2009], which considers H-oil transfer and oil delivery processes via two pipelines. For decreasing operation cost, to transfer H-oil of a type as much as possible from storage tanks once to the refinery by an individual setup, so the objective function of our model is to minimizing the number of switches between H-oil transfer operation decision and L-oil transfer one in the same pipeline which can increase the cost of H-oil transfer. This model is a MINLP model, so it cannot be solved using CPLEX easily. We can abandon bilinear constraints to form a MILP model and use two-stage MILP-NLP heuristic algorithm to solve this model and get a satisfactory optimality gap.

In the next section, this paper briefly introduces refinery processes addressed in it, as well the short-

term scheduling problem are presented. Then, Section 3 develops the continuous-time single-operation sequencing (SOS) formulation considering high-fusion-point oil and two-pipeline transfer. Section 4 proposes an efficient approach to solve the problem. The applications of the proposed method are illuminated by an actual industrial in Section 5. Finally, conclusion of this paper is drawn in Section 6.

## **THE PROCESSES AND THEIR SCHEDULING PROBLEM**

In this section crude oil operations are briefly introduced and its short-term scheduling problem is defined.

### **The processes of crude oil operations**

Oil refinery processes involving usually three phases are illustrated in Figure 1: (1) crude oil operations; (2) production; and (3) product delivery. This work aims at crude oil operation short-term scheduling in operating a refinery, which belongs to the hardest scheduling problems. During the crude oil operation phase, crude is conveyed to a harbor which is close to the refinery by means of vessels and offloaded into storage tanks. Then it is transferred to refinery's charging tanks by way of pipelines. It is charged into crude distillation units for distillation from the charging tanks. A variety of crude oil types need refining in a refinery among which is H-oil. The fusion point of H-oil usually is higher than 30 °C and at normal temperature its state is solid. Usually, to facilitate unloading, storing, transferring crude oil through a submarine pipeline for crude oil vessels, a location is built. Location #1 in Figure 1 is exploited to illustrate such a location. Unfortunately, the submarine temperature is invariably under 30 °C and such an undersea pipeline cannot permit H-oil to transfer through it. Therefore, a refinery processing H-oil have no choice but to utilize a jetty which is sufficiently close to the land such that a grounded can be constructed. This kind of jetty is usually small-scale and can only anchor small tankers. In Figure 1, we apply Location #2 to indicate this kind of facility. Using small tankers leads to the relatively high transfer cost, so L-oil is delivered to Location #1 and only H-oil is to Location #2.

Unless the same type of crude oil is in storage tank, refinery normally unloads crude oil into an empty one. Between being charged and discharging crude oil operations of a tank, it must keep the crude oil in it sojourn for a required period of time, named oil residency time (RT). Sometimes, while transferring

crude oil via a pipeline to charging tanks, crude oil of diverse types have to be blended, which degrade the quality of crude oil. Crude oil always fills a pipeline and the pipeline cannot be exhausted. At each location, refinery processes multiple crude oil types, from storage tanks to charging ones which are transported through a pipeline. Therefore, switching from one type of oil to another now and then is required. So, multiple segments of crude oil with diverse types may sojourn in or run across an identical pipeline. A charging tank can be fed by one pipeline at a time. Besides, any tanks cannot simultaneously receive and send oil which includes storage tanks and charging tanks.

Before Pipeline #2 being charged it should be heated in case H-oil becomes frozen throughout the course of its transfer. Only the flowing of hot L-oil in charging tanks through Pipeline #2 backwards to storage tanks at Location #2 can heat it. When it is sufficiently hot, the hot H-oil can be transferred from storage tanks at Location #2 where it is contained to charging tanks. When H-oil is discharged in Pipeline #2, it has to be kept fluid; otherwise ambient temperature would make the pipeline cooling and the H-oil within it would also be cooled so as to become solid. This requirement is referred to as H-oil transfer constraint. Thus, after the last H-oil parcel needing transferring is emitted into the pipeline completely, it is necessary to emit another L-oil parcel in the storage tanks at Location #2 into the pipeline to eject the H-oil within it totally. It is required that Location #2 should have sufficient L-oil and the refinery's side should possess adequate space of charging tank. Namely, in contrast with a one-pipeline system, a two-pipeline system transferring H-oil on which more constraints are imposed requires more accessory operations and resources (storage tanks and charging ones), which complicates the process in the extreme and leads to a quite academically challenging scheduling problem. As is also proverbial that the cost of setup for transferring H-oil is greatly high. Therefore, in scheduling operations of crude oil in such a system, it is very much desired to transfer as much H-oil as possible by individual setup. Notice that Pipeline #1 only permit unidirectional oil flows to run across, namely, from storage tanks to charging ones; while Pipeline #2 permits bidirectional oil flows to go through. There are novel questions which do not exist in one-pipeline systems, for example, under what circumstances and when to transfer H-oil, as well how to make the operations for both pipelines in concert with each other.

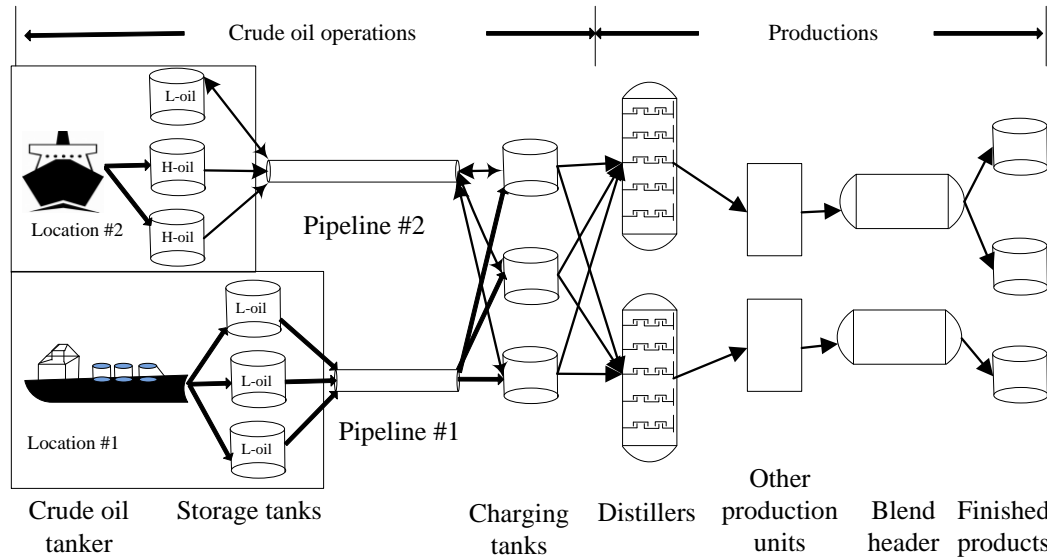


Figure 1 A 2-pipeline oil refinery process

In general, the following resource and process constraints must be respected by a short-term schedule. The former includes: (1) the capacity of storage tanks and charging tanks, as well the limited number of them; (2) the limited flow rate of crude oil offloading and transferring through pipelines; and (3) the available amount of different crude oil types in coming vessels and tanks. The latter includes: (1) unless a maintenance is needed, a distiller should be kept uninterrupted working at every moment; (2) a distiller can be charged by at least one charging tank at all times; (3) charging and discharging operation of a tank cannot be executed simultaneously; (4) for any tank, after being charged RT constraint must be satisfied; and (5) transferring H-oil constraint must be satisfied.

### Short-term scheduling

For crude oil operations, resources include docks for crude vessels, charging tanks, storage tanks, and two pipelines. There are four kinds of operations: unloading operation of crude from vessels to storage tanks, crude transportation from storage tanks to charging ones via pipelines, crude oil transfer from charging tanks to storage ones in Location #2 via a pipeline; and crude oil in charging tanks feeding distillation units (CDU). The short-term scheduling problem of crude oil operations is a procedure to define all operations to be performed and then sequence them by assigning the resources to these operations for a scheduling horizon enduring through a week, ten days, or even longer. To do so, at the beginning, one knows the initial state information of the system only. It includes 1) the current inventory of crude oil and crude oil types in storage and charging tanks; 2) the arrival time of marine tankers, crude oil types and volume in them; and 3) operation state of each production device.

For the discussion of this paper, without loss of generality, the following assumptions are made: 1) just one berth for vessel unloading is available at the

docking station; 2) a tank feeds at most one CDU at a time; and at most one charging tank can charge a CDU at a time.

In scheduling crude oil operations, there are diverse objectives and it is a typical multi-objective optimization problem [Wu *et al.*, 2005]. To transfer crude oil in storage tanks into charging ones by a pipeline needs switching among different types of oil, resulting in unnecessary oil mixing. Also, when there is switching between two charging tanks to feeding a CDU, there is a set point regulation process. As a consequence, both switches result in a cost. Moreover, when any of such switches occurs, a setup is necessary. Such a setup is not only time consuming but also hazardous in the sense of security [Saharidisa *et al.*, 2009]. Last but not least, the transporting H-oil setup cost is very high. Therefore, it is crucial to minimize the number of switches for both of them as is done in [Saharidisa *et al.*, 2009] as well as the number of setup for H-oil transfer, which is the objective for our model.

As is discussed above, a short-term crude oil operations schedule is constituted by a sequence of crude oil delivering operations. To solve the above schedule problem, we must answer the following two questions. One is when an operation should happen, the other is what and how it should be executed. We should make a decision for every operation to occur for the sake of answering the questions. In order to represent a short-term schedule in detail, an operation decision (OD) is defined as follows.

**Definition 2.1:** An operation decision is defined as  $OD = (S, D)$ , where  $S$  is the source from which the crude oil comes, and  $D$  is the destination crude oil delivered to.

Generally, three kinds of ODs are in crude oil operations: 1) oil in tankers unloading OD to storage tanks; 2) oil in storage tanks transportation OD to charging tanks; and 3) oil in charging tanks feeding OD to CDUs. Because of the limitation of resources, many operations should be performed one after

another. Hence, the non-overlapping requirement for two ODs  $v$  and  $w$  becomes one of the most general constraints of crude oil scheduling problems. This requirement is stated as follows.

Definition 2.2: Assume that ODs  $v$  and  $w$  are to be performed in  $[t_{v1}, t_{v2}]$  and  $[t_{w1}, t_{w2}]$ , respectively, and  $[t_{v1}, t_{v2}] \cap [t_{w1}, t_{w2}] = \emptyset$  should hold. Then, this is called the non-overlapping requirement.

A short-term crude oil operations schedule consists of a sequence of ODs. By the priority-slot-based method presented in [Mouret *et al.*, 2009], to describe a schedule with a continuous-time formulation is to sequence these ODs by using a priority slot sequence. A priority slot  $i$  is defined as a point  $i$  on the time coordinate. Slot  $i$  is said to have a more preferential scheduling priority than slot  $j$  with slots  $i$  and  $j$  being non-overlapping, if  $i$  is placed earlier than  $j$  on the time coordinate. Such a relation is denoted as  $j > i$ , or  $i < j$ . By the priority-slot-based method, to formulate a scheduling problem, each priority slot is assigned to just one definite OD. In this way, the priority slot number is equal to the total OD number which will be performed during the whole scheduling horizon. The priority slot sequence corresponds to the sequence of the ODs. By this means, the key is to decide the priority slot number that is required to be known in advance.

Assume that two ODs  $v$  and  $w$  which are non-overlapping and assigned to priority slots  $i$  and  $j$  with  $i < j$ . Let  $S_{iv}$  and  $S_{jw}$  be the start time of slots  $i$  and  $j$ , and  $D_{iv}$  and  $D_{jw}$  be their operation durations, respectively. Since OD  $v$  has a higher priority than  $w$ ,  $w$  is able to start only after the completion of  $v$ , i.e., we have

$$S_{iv} + D_{iv} \leq S_{jw} \quad (2.1)$$

With this precedence relationship, given a sequence of ODs, a schedule obtained is feasible only if (2.1) is satisfied for any pair of non-overlapping ODs. In the meanwhile, with this priority-slot-based modeling strategy, different schedules can be obtained by ordering the ODs according to their start time. An example in a refinery scheduling from [Lee *et al.*, 1996] can be used to show this modeling strategy. The configuration of the system and relative data can be referred to [Mouret *et al.*, 2009]. The ODs and their priority-slots assignment shown in Figure 2 present the optimal schedule. This schedule can be denoted as the sequence of ODs 7683513762, where a number denotes an OD. For example, number 7 represents an OD of Feeding 7. Thus, for this schedule, there are two ODs of transfer 6 and they are assigned to two non-overlapping slots 2 and 9 such that the non-overlapping constraint is satisfied.

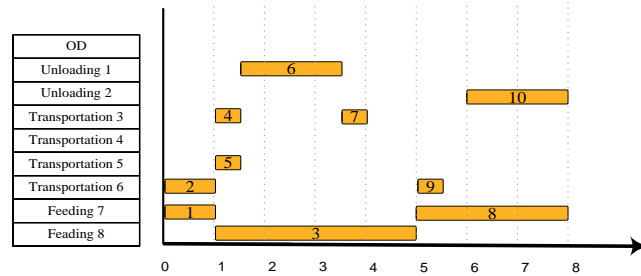


Figure 2 The optimal schedule for the example

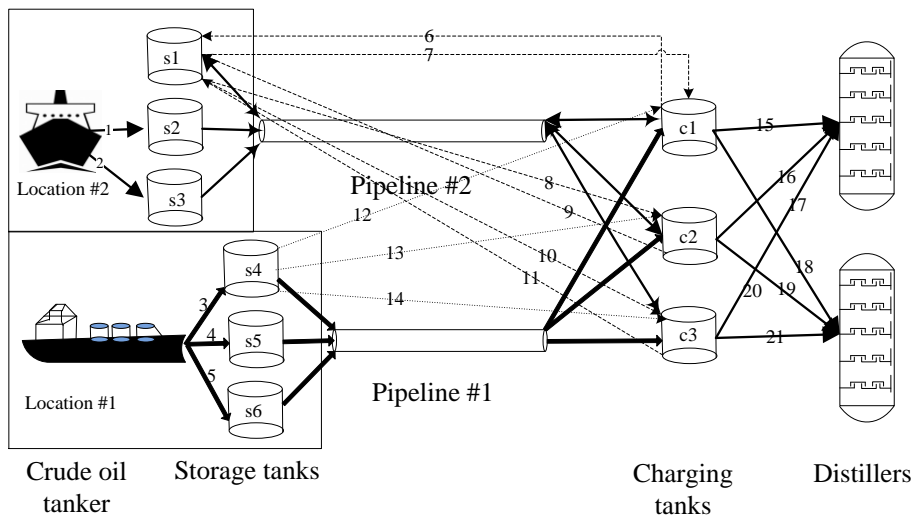


Figure 3 The refinery configuration for modeling illustration

## PROBLEM FORMULATION

This section presents the priority-slot-based continuous-time formulation for the short-term crude oil operations scheduling problem with oil residency time constraints, high-fusion-point oil and two pipelines transfer being taken into account. For the purpose of readability, a process of crude oil operations shown in Figure 3 is used to illustrate the modeling process. Here solid lines denote actual crude oil flow directions, and dashed ones are used to express simplified directions. For the sake of simplicity, some dashed lines are omitted. First, we present the notation for the model.

### Sets and parameters

$T = \{1, 2, \dots, n\}$ : Set of priority-slots  
 $W$ : Set of the  $n$  ODs for a schedule:  
 $W = W_U \cup W_T \cup W_D$  ( $W = \{1-21\}$ ) for the system shown in Figure 3)  
 $W_{UH}$ : Set of unloading ODs of H-oil vessels  
 $W_{UL}$ : Set of unloading ODs of L-oil vessels  
 $W_U = W_{UH} \cup W_{UL}$   
 $W_U \subset W$ : Set of unloading ODs ( $W_U = \{1-5\}$  for the system shown in Figure 3)  
 $W_{TF}$ : Set of oil transfer ODs from storage tank to charging tank ( $W_{TF} = \{7, 8, 10, 12-14\}$  for the system shown in Figure 3)  
 $W_{TO}$ : Set of oil transfer ODs from charging tank to storage tank at site #2 ( $W_{TO} = \{6, 9, 11\}$  for the system shown in Figure 3)  
 $W_T = W_{TF} \cup W_{TO}$   
 $W_T \subset W$ : Set of oil transfer ODs ( $W_T = \{6-14\}$ )  
 $W_D \subset W$ : Set of oil feeding ODs ( $W_D = \{15-21\}$ )  
 $R_{VH} \subset R$ : Set of H-oil vessels at site #2  
 $R_{VL} \subset R$ : Set of L-oil vessels at site #1  
 $R_V = R_{VH} \cup R_{VL}$   
 $R = R_V \cup R_S \cup R_C \cup R_D$ : Set of resources  
 $r_{\text{special}}$ : Storage tank at site #2 which is used to eject the H-oil in pipeline2 after H-oil transfer finishing H-oil transportation  
 $R_{SH} \subset R$ : Set of H-oil storage tank at site #2  
 $R_{SL} \subset R$ : Set of L-oil vessels at site #1  
 $R_S = R_{SH} \cup R_{SL} \cup \{r_{\text{special}}\}$   
 $R_S \subset R$ : Storage tanks set  
 $R_C \subset R$ : Charging tanks set  
 $R_D \subset R$ : CDUs set  
 $I_r \subset W$ : Entry transfer ODs set of resource  $r$   
 $O_r \subset W$ : Exit transfer ODs set of resource  $r$   
 $C$ : Set of crude oil types  
 $C_H$ : Set of H-oil types  
 $C_L$ : Set of L-oil types  
 $C = C_H \cup C_L$   
 $H$ : Scheduling horizon  
 $V_{p1}$ : Capacity of L-oil pipeline1  
 $V_{p2}$ : Capacity of H-oil pipeline2  
 $V_{\text{hot}}$ : Indispensable hot L-oil volume which should run across Pipeline #2 to heat it  
 $RT$ : Oil residency time

$LV_v$  and  $UV_v$ : Lower and upper bounds of the volume delivered by OD  $v$  and generally  $LV_v = 0$  for an OD except that an unloading OD is required to unload all the oil of a type in a vessel once, in this case,  $LV_v = UV_v$

$LF_v$  and  $UF_v$ : Minimal and maximal flow rates permissive for OD  $v$ ,  $v \in W_U \cup W_D$

$LF_{vh}$  and  $UF_{vh}$ : Minimal and maximal flow rates permissive for Pipeline #2

$LF_{vl}$  and  $UF_{vl}$ : Minimal and maximal flow rates permissive for Pipeline #1

$S_r$ : Arrival time of vessel  $r$

$F_{\text{pipeline2}}$ : Transfer flow rate of Pipeline #2

$F_{\text{pipeline1}}$ : Transfer flow rate of Pipeline #1

$LC_r$  and  $UC_r$ : Capacity limit of tank  $r \in R_S \cup R_C$

$LD_r$  and  $UD_r$ : Lower and upper bounds of demand on crude oil to be delivered from charging tank  $r \in R_C$  during the scheduling horizon

### Variables

Binary assignment variables:  $Z_{iv} \in \{0, 1\}$ ,  $i \in T$  and  $v \in W$ . If priority-slot  $i$  can accommodate OD  $v$ ,  $Z_{iv} = 1$ , and otherwise  $Z_{iv} = 0$ .

Continuous time variables:  $S_{iv} \geq 0$  and  $D_{iv} \geq 0$ ,  $i \in T$  and  $v \in W$ . If priority-slot  $i$  can accommodate OD  $v$ ,  $S_{iv}$  is the beginning time of OD  $v$ , and otherwise  $S_{iv} = 0$ . If priority-slot  $i$  can accommodate OD  $v$ ,  $D_{iv}$  is the duration of OD  $v$ , and otherwise  $D_{iv} = 0$ .

Operation variables:  $V_{iv} \geq 0$  and  $V_{ivc} \geq 0$ ,  $i \in T$ ,  $v \in W$ , and  $c \in C$ , where  $V_{iv}$  is the entire volume of crude oil delivered by OD  $v$  if priority-slot  $i$  can accommodate it, and otherwise  $V_{iv} = 0$ .  $V_{ivc}$  is the crude oil volume of type  $c \in C$  delivered by OD  $v$  if priority-slot  $i$  can accommodate it, and otherwise  $V_{ivc} = 0$ .

Resource variables:  $L_{ir}$  and  $L_{irc}$ ,  $i \in T$ ,  $r \in R$ , and  $c \in C$ , where  $L_{ir}$  is the entire aggregated volume of crude oil in  $r \in R_S \cup R_C$  at the beginning of slot  $i$ .  $L_{irc}$  is the aggregated volume of crude oil type  $c$  in  $r \in R_S \cup R_C$  at the beginning of slot  $i$ .

Auxiliary continuous variables:  $x > 0$ .

With the notation given above, we can present our formulation for the short-term crude oil operations scheduling problem with oil residency time constraints, high-fusion-point oil and two pipelines transfer being taken into account by using the priority-slot-based method given in [Mouret *et al.*, 2009]. First, we present the constraints as follows.

### Constraint for assigning ODs to priority slots

One OD should be assigned to exactly one priority slot. This is stated by the following constraint.

$$\sum_{v \in W} Z_{iv} = 1 \quad (1)$$

### Variable constraints

Only priority slot  $i \in T$  accommodates OD  $v \in W$ , do  $S_{iv} \geq 0$  and  $D_{iv} \geq 0$  hold. Thus, the following constraint should be satisfied.

$$S_{iv} + D_{iv} \leq H \times Z_{iv}, \quad i \in T \text{ and } v \in W \quad (2)$$

When OD  $v \in W$  is assigned to slot  $i \in T$ , the volume of oil delivered by this OD should be no more than its upper bound.

$$V_{iv} \leq UV_v \times Z_{iv}, \quad i \in T \text{ and } v \in W \quad (3)$$

When OD  $v \in W$  is assigned to slot  $i \in T$ , the volume of oil delivered by this OD should be no less than its lower bound.

$$V_{iv} \geq LV_v \times Z_{iv}, \quad i \in T \text{ and } v \in W \quad (4)$$

The volume of crude oil delivered by an OD should be equal to the sum of the volume of all the oil types delivered by the OD. Because crude oil mixing is prohibited,  $V_{iv}$  contains only one type of crude oil.

$$\sum_{c \in C} V_{ivc} = V_{iv}, \quad i \in T, \quad v \in W \text{ and } c \in C \quad (5)$$

The following two constraints present the material balance in a tank or a vessel at the beginning of a slot.

$$L_{ir} = L_{0r} + \sum_{j \in T, j < i} \sum_{v \in I_r} V_{jv} - \sum_{j \in T, j < i} \sum_{v \in O_r} V_{jv}, \quad i, j \in T, \quad r \in R \quad (6)$$

$$L_{irc} = L_{0rc} + \sum_{j \in T, j < i} \sum_{v \in I_r} V_{jvc} - \sum_{j \in T, j < i} \sum_{v \in O_r} V_{jvc}, \quad i, j \in T, \quad r \in R \text{ and } c \in C \quad (7)$$

### Cardinality constraint

The unloading of crude oil in a crude oil vessel should be completed by a single OD. Thus, we have the following constraint.

$$\sum_{i \in T} \sum_{v \in O_r} Z_{iv} = 1, \quad r \in R_V \quad (8)$$

### Unloading sequence constraint

By assumption, site #1 and site #2 has only one dock and it can berth only one vessel with L-oil and H-oil at a time, respectively. Hence, only one vessel can be unloaded at a time and the following constraint should be satisfied.

$$\sum_{j \in T, j < i} \sum_{v \in O_{r2}} Z_{jv} + \sum_{j \in T, j \geq i} \sum_{v \in O_{r1}} Z_{jv} \leq 1, \quad (9)$$

$$\sum_{j \in T, j < i} \sum_{v \in O_{r2}} Z_{jv} + \sum_{j \in T, j \geq i} \sum_{v \in O_{r1}} Z_{jv} \leq 1, \quad (10)$$

$i \in T, \quad i \neq 1, \text{ and } r1, r2 \in R_{VH}$

$i \in T, \quad i \neq 1, \text{ and } r1, r2 \in R_{VL}$

### Non-overlapping constraints

With only one vessel being unloaded, two unloading ODs of an identical vessel must not overlap. Constraint (11) and (12) make the vessels docking at site #1 and #2 respect the above law, respectively.

$$\sum_{v \in W_{UH}} (S_{iv} + D_{iv}) \leq \sum_{v \in W_{UH}} S_{jv} + H \times (1 - \sum_{v \in W_{UH}} Z_{jv}), \quad (11)$$

$i, j \in T \text{ and } i < j$

$$\sum_{v \in W_{UL}} (S_{iv} + D_{iv}) \leq \sum_{v \in W_{UL}} S_{jv} + H \times (1 - \sum_{v \in W_{UL}} Z_{jv}), \quad (12)$$

$i, j \in T \text{ and } i < j$

A tank must be prevented from being charged and discharging simultaneously. Thus, for storage tanks and charging ones, the following constraints should be satisfied.

$$\sum_{v \in I_r} (S_{iv} + D_{iv}) \leq \sum_{v \in O_r} S_{jv} + H \times (1 - \sum_{v \in O_r} Z_{jv}), \quad (13)$$

$i, j \in T, \quad i < j, \quad r \in R_S \cup R_C$

$$\sum_{v \in O_r} (S_{iv} + D_{iv}) \leq \sum_{v \in I_r} S_{jv} + H \times (1 - \sum_{v \in I_r} Z_{jv}), \quad (14)$$

$i, j \in T, \quad i < j, \quad r \in R_S \cup R_C$

A tank may charge only one tank at a time. Constraints (15) and (16) restraint storage tanks at Location #1 and charging ones which can do reverse transfer to abide by the above rule. Constraints (17) and (18) enforce the storage tanks in site #2 to respect the same rule.

$$\sum_{v \in O_r} (S_{iv} + D_{iv}) \leq \sum_{v \in O_r} S_{jv} + H \times (1 - \sum_{v \in O_r} Z_{jv}), \quad (15)$$

$i, j \in T, \quad i < j, \quad r \in R_{SL} \cup R_C$

$$\sum_{v \in O_r} (S_{iv} + D_{iv}) + V_{P1}/F_{pipeline1} \leq H, \quad (16)$$

$i \in T \text{ and } r \in R_{SL} \cup R_C$

$$\sum_{v \in O_r} (S_{iv} + D_{iv}) \leq \sum_{v \in O_r} S_{jv} + H \times (1 - \sum_{v \in O_r} Z_{jv}), \quad (17)$$

$i, j \in T, \quad i < j, \quad r \in R_{SH} \cup R_C$



$$\sum_{v \in O_r} (S_{iv} + D_{iv}) + V_{P2}/F_{pipeline2} \leq H, \quad (18)$$

$$i \in T \text{ and } r \in R_{SH} \cup R_C$$

A CDU can be charged by only one charging tank at a time.

$$\sum_{v \in I_r} (S_{iv} + D_{iv}) \leq \sum_{v \in I_r} S_{jv} + H \times (1 - \sum_{v \in I_r} Z_{jv}), \quad (19)$$

$$i, j \in T, i < j \text{ and } r \in R_D$$

To prohibit schedules where transportation is being repeatedly executed at a time, the model includes Constraint (20).

$$S_{iv} + D_{iv} \leq S_{jv} + H \times (1 - Z_{jv}), \quad (20)$$

$$i, j \in T, i < j \text{ and } v \in W$$

#### Continuous distillation constraint

A CDU should operate without interruption. To do so, for each CDU, there is an OD for feeding it, or the following constraint should be satisfied.

$$\sum_{i \in T} \sum_{v \in I_r} D_{iv} = H, \quad r \in R_D \quad (21)$$

#### Resource availability constraint

The unloading of a crude oil vessel can begin only after the arrival of the vessel at the port. Let  $S_r$  be the arrival time of vessel  $r$ , and then the following constraint should be satisfied.

$$S_{iv} \geq S_r \times Z_{iv}, i \in T, r \in R_V \text{ and } v \in O_r \quad (22)$$

#### Flow rate constraint

The flow rate of an oil transfer OD  $v$  from vessel to storage tank or from charging tank to CDU should be bounded by  $LF_v$  and  $UF_v$ , i.e.,

$$LF_v \times D_{iv} \leq V_{iv} \leq UF_v \times D_{iv}, \quad (23)$$

$$i \in T \text{ and } v \in W_{II} \cup W_D$$

The flow rate of an oil transfer OD  $v$  from storage tank at site #2 to charging tank should be bound by  $LF_{vh}$  and  $UF_{vh}$ , i.e.,

$$LF_{vh} \leq F_{pipeline2} \leq UF_{vh} \quad (24)$$

The flow rate of an oil transfer OD  $v$  from storage tank at site #1 to charging tank should be bound by  $LF_{vl}$  and  $UF_{vl}$ , i.e.,

$$LF_{vl} \leq F_{pipeline1} \leq UF_{vl} \quad (25)$$

#### Tank capacity constraint

The volume of oil in tank  $r$  must remain in a permissive interval  $[LC_r, UC_r]$ . Let  $L_{0r}$  be the original volume of oil in  $r$  that contains  $L_{0rc}$  of crude oil type  $c$ . Because simultaneous charging and discharging of a tank are not permissive, the following constraints describe this requirement.

$$LC_r \leq L_{ir} \leq UC_r, i \in T \text{ and } r \in R_S \cup R_C \quad (26)$$

$$0 \leq L_{irc} \leq UC_r, \quad (27)$$

$$i \in T, c \in C, \text{ and } r \in R_S \cup R_C$$

$$LC_r \leq L_{0r} + \sum_{i \in T} \sum_{v \in I_r} V_{iv} - \sum_{i \in T} \sum_{v \in O_r} V_{iv} \leq UC_r, \quad r \in R_S \cup R_C \quad (28)$$

$$0 \leq L_{0rc} + \sum_{i \in T} \sum_{v \in I_r} V_{ivc} - \sum_{i \in T} \sum_{v \in O_r} V_{ivc} \leq UC_r, \quad r \in R_S \cup R_C \text{ and } c \in C \quad (29)$$

#### Demand constraint

Demand constraint defines the upper and lower limits  $UD_r$  and  $LD_r$  on the total volume of crude oil transported into CDU  $r$  during the scheduling horizon.

$$LD_r \leq \sum_{i \in T} \sum_{v \in I_r} V_{iv} \leq UD_r, \quad r \in R_D \quad (30)$$

#### Storage tanks residency time constraint

$$\sum_{v \in I_r} (S_{iv} + D_{iv} + RT \cdot Z_{iv}) \leq (\sum_{v \in O_r} S_{jv} + (H + RT) \cdot (1 - \sum_{v \in O_r} Z_{jv})), \quad (31)$$

$$i, j \in T, i < j, r \in R_S$$

#### Charging tanks residency time constraint

$$\sum_{v \in O_r} (S_{iv} + H \cdot (1 - Z_{iv})) \geq (\sum_{v \in I_r} (S_{jv} + D_{jv}) + RT \cdot \sum_{v \in I_r} Z_{jv}), \quad (32)$$

$$i, j \in T, i > j, r \in R_C$$

#### Pipeline fullness in last time slot constraint

The pipeline should be full of crude oil at the scheduling horizon end.

$$\sum_{j \in T, j < i} \sum_{v \in O_r} V_{jv} - \sum_{r \in R_C} L_{ir} - V_{P1} \geq \sum_{r \in R_C} \sum_{i \in T} \sum_{v \in O_r} V_{iv}, \quad (33)$$

$$i = |T| \text{ and } r \in R_S$$

$$\sum_{j \in T, j < i} \sum_{v \in O_r} V_{jv} - \sum_{r \in R_C} L_{ir} - V_{P2} \geq \sum_{r \in R_C} \sum_{i \in T} \sum_{v \in O_r} V_{iv}, \quad (34)$$

$$i = |T| \text{ and } r \in R_S$$

#### Heating pipeline2 and H-oil transfer beginning constraint

$$\left\{ \begin{array}{l} Z_{kv} = Z_{jv'} = Z_{mv''} \\ v \in O_r, r = r_V \quad v' \in W_{TO} \quad v'' \in O_r, r \in R_{SH} \\ L_{jrc} \geq V_{hot} \cdot Z_{jv'} \\ D_{jv'} = V_{hot}/F_{pipeline2} \cdot Z_{jv'} \\ S_{kv} = (S_{jv'} + D_{jv'}) \cdot Z_{jv'} \\ D_{kv} = V_{P2}/F_{pipeline2} \cdot Z_{jv'} \\ S_{mv''} = (S_{kv} + D_{kv}) \cdot Z_{jv'} \\ v'' \in O_r, r \in R_{SH} \\ m, k, j \in T, j < k < m \text{ and } c \in C/C_L \end{array} \right. \quad (35)$$

If  $Z_{jv'} v' \in W_{TO}$ ,  $Z_{kv} v \in O_r, r = r_V = 1$  and there is a charging tank transport hot L-oil to heat pipeline2



in preparation for H-oil transfer and  $L_{jrc} \geq V_{hot} \cdot Z_{jv'}$  assures that its volume  $L_{jrc}$  is greater than  $V_{hot}$ ,  $D_{jv'} = V_{hot}/F_{pipeline2}$ . And then  $Z_{kv,v \in O_r, r=r_v}=1$  compels the L-oil in  $r_v$  to be ejected to some charging tank. H-oil begins to be transported at the same time  $Z_{mv'',v'' \in O_r, r \in R_{SH}} = S_{kv} + D_{kv}$ . Otherwise, there is no L-oil reverse transfer, and H-oil transfer will be impossible. Constraint (35) implies this process. Constraint (36) shows the volume of  $r_{special}$  in time slot  $i$ .

$$L_{irc} = L_{0rc} + \sum_{j \in T, j < i} \sum_{v \in I_r} V_{hot} - \sum_{j \in T, j < i} \sum_{v \in I_r} V_{P2}, i \in T, c \in C, r = r_{special} \quad (36)$$

### H-oil eject constraint

Constraint (37) assures that all the high fusion point crude oil pumped into the pipeline must be ejected from it. The values of two flanks of the first equation in constraint (37) have two cases; one is  $S_{jv'} - E_{iv} > 0$  and  $\sum_{v' \in O_r, r=r_{special}} Z_{v'} = 0$  which is impossible because of non-overlapping constraint, the other is  $S_{jv'} - E_{iv} = 0$  and  $\sum_{v' \in O_r, r=r_{special}} Z_{v'} = 1$

which means that the end time of transferring high fusion point oil is the beginning time to transfer low fusion point oil from  $r_{special}$  to eject high fusion point oil in pipeline #2. The second equation  $V_{iv'c} = V_{P2}$  decides the adequate volume of low fusion point oil to finish the ejection operation.

$$\begin{cases} S_{jv'} - E_{iv} = (1 - Z_{kv'}) \times x \\ Z_{kv'} \times V_{iv'c} = V_{P2} \times Z_{kv'} \\ i, j \in T, i < j, v' \in O_r, r = r_{special} \end{cases} \quad (37)$$

### Objective function

The objective is to minimize the number of reverse transfer because a reverse transfer operation corresponds to an H-oil setup and minimizing the number of reverse transfer means minimizing the total setup cost for H-oil transportation. Therefore, the objective function is as follows.

$$J = \sum_{i \in T} \sum_{v \in W_{TO}} Z_{iv} \quad (38)$$

Based on the discussion above, the short-term crude oil operations scheduling problem with oil residency time constraints, high-fusion-point oil and two pipelines transfer can be formulated as the following mathematical programming problem.

$$\text{Problem P1: Minimize } J = \sum_{i \in T} \sum_{v \in W_{TO}} Z_{iv}$$

Subject to: constraints (1)-(37).

With the formulations for the scheduling problem developed above, we discuss how to solve the problem next.

## SOLUTION METHOD

As mentioned above, Problem P1 is a mixed integer non-linear programming model (MINLP). Before solving this model, we have to determine the number of priority slots. We use  $P1(n)$  to denote Problem P1 with priority slots number  $n$ . The previous researchers usually adopt the following methods to determine the optimal time slot or event points: first assume a time slot number  $n$ , solve model  $P1(n)$ ,  $n \leftarrow n+1$ , and then repeat solving  $P1(n)$  until the value of objective function does not change. We use bisearch method to improve the above algorithm and greatly reduce the solution times of  $P1(n)$  in another article.

Due to the existence of bilinear constraints in our MINLP model, the solution space of this model is non-convex and it is difficult to obtain the global optimal solution. The commercial software DICOPT using the outer approximation algorithm can be exploited to solve non-convex MINLP model, but may converge to a suboptimal solution; and as to another commercial software BARON using branch and bound search algorithm of the commercial software BARON, although it can find the global optimal solution, but highly time-consuming. The main purpose of this paper is to model and then verify the validity of the model, so we use a simple two-stage MILP-NLP algorithm. In first stage, we solve the MILP relaxation of the original model (by removing bilinear constraints), then fix binary variables in original model using the values of binary variables in the solution obtained in first stage, get a NLP model and solve it using the solution found in the first stage as the starting point. Although the solution obtained in the second stage may not be the optimal solution of the original model, we can use the MILP solution in the first stage as an upper bound, the NLP solution in the second stage as a lower bound to estimate the optimality gap.

## INDUSTRIAL CASE STUDY

We adopt an actual application case from a large-scale Chinese refinery as study case which is presented in [Wu *et al.*, 2016]. We simplify this case in order to test our model. The scheduling horizon is 15.5 days. The refinery has two distillers DS1-DS2 where DS2 is capable of processing H-oil. At the start 6 charging tanks TK1-TK6 are available for feeding distiller for the case problem, TK1-TK3 serve DS2, TK4-TK6 serve DS1, respectively. The original charging tank state is shown in Table 1 in [Wu *et al.*, 2016]. The maximal Pipeline #1 flow rate is 625 tons/h, as well the minimal and maximal Pipeline #2 flow rates are 420 tons/h and 625 tons/h, respectively. Charging flow rates of the distillers are 250 tons/h, 334 tons/h for DS1 and DS2, respectively. The demands of the CDUs are 93,055 tons, 121,285 tons for DS1 and DS2, respectively. Furthermore, as is known that the value of  $V_{hot}$  is 25,000 tons, the pipeline #2

capacity  $V_{p2} = 18,000$  tons, and  $RT = 4$  h. At Location #2, there is 134,000 tons of H-oil #11 in tanks, the capacity of two tanks. 24,000 tons of oil

#4, 136,000 tons of oil #8, 118,000 tons of oil #9, and 80,000 tons of oil #10 are at Location #1.

Table 1 The initial state for the charging tanks [Wu *et al.*, 2016]

Charging tank	Capacity (ton)	Type of oil filled	Volume of oil in the tank (ton)
TK1	20,000		
TK2	34,000	Oil #1	16,000
TK3	34,000	Oil #2	26,000
TK4	25,000		
TK5	30,000	Oil #3	25,000
TK6	34,000	Oil #5	15,000
TK7	20,000	Oil #4	17,000

In the next stage, crude oil #11 needs processing by DS2 after the oil within TK2 and TK3 being processed by it. Therefore, it is expected to transfer the oil #11 at Location #2 into the charging tanks totally, or the volume of H-oil required being delivered 134,000 tons. 25,000 tons of hot L-oil in TK5 is ready for running across Pipeline #2 and heating it in order to transfer H-oil via Pipeline #2.

On the basis of the above system configuration we make the following assumptions.. We assume that there are three storage tanks S1-S3 at site #2 which hold 3# L-oil 18,000 tons, #11 H-oil 64,000 tons and #11 H-oil 70,000 tons, respectively and transport crude oil through pipeline2. At site #1, four storage tanks S4-S7 hold #4 L-oil 24,000 tons, #8 L-oil 136,000 tons, #9 L-oil 118,000 tons and

#10 L-oil 80,000 tons, respectively and transport crude oil through pipeline1. There are 6 charging tanks C1-C6 in refinery, C1-C3 serve DS2, C4-C6 serve DS1, respectively. In addition, because the demand of DS1 is 93,055 tons and the total H-oil is 134,000 tons, storage tanks at site #1 need not transport L-oil to C1-C3. So, the number of all operations in the system is 21. Based on the above assumptions, we establish MINLP model with priority-slot number 21. We use Algorithm 1 to solve the MINLP model, and get the detailed scheduling in Figure 4 and Figure 5, respectively. Figure 4 shows the detailed schedule with volume and flow rate for tank charging via Pipeline #2, and Figure 5 shows the detailed schedule with volume and flow rate for tank charging via Pipeline #1.

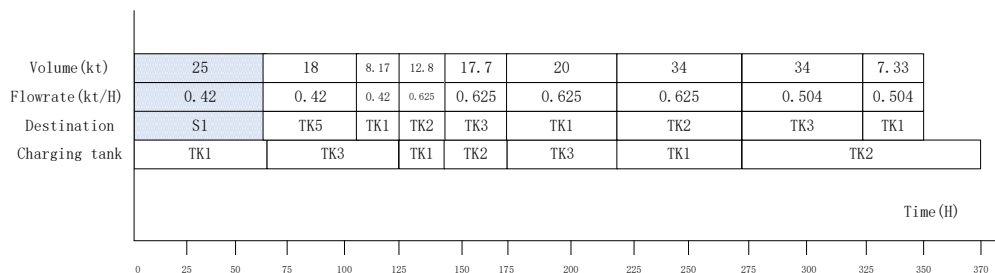


Figure 4 The detailed schedule with volume and flow rate of tank charging via Pipeline #2

As is shown in Figure 4, under the condition that three charging tanks feed DS2, we get a schedule, in which all H-oil at Location #2 can be transferred, and we must use the L-oil in S1 at Location #2 to eject the H-oil. At the same time, scheduling result shows that the H-oil amount that can be transferred and the setup cost of transfer H-oil is greatly sensitive to the system initial state, especially the flow rate of pipeline2. When the flow rate lower bound of pipeline2 is enough large, the H-oil

transfer cannot continuous which need more than one setup, so this increases the cost of the H-oil transfer. The shade one in Fig 4 denotes the L-oil in a charging tank flow through pipeline2 to a storage tank. The shade one in Fig 5 denotes the oil flow from a pipeline2 to charging tank TK5. The latter shade is smaller than the former because a part of L-oil has been transported into storage tank in Site #2.

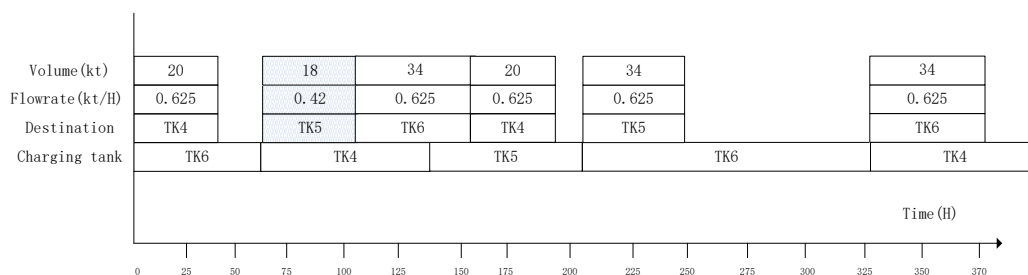


Figure 5 The detailed schedule with volume and flow rate for tank charging via Pipeline #1

Note that both starting and ending of an event in the resulting schedule can happen at any time. Therefore, discrete-time representation based mixed integer nonlinear programming model can be applied in order to avoid nonlinear constraints. Although our model does not require uniform time slot which can create thousands of discrete variables, when the number of priority-slot is 40 and the number of operation is 40, our model yet has more than 1600 discrete variables and hard to solve using DICOPT solver directly. But we can use DICOPT solver combined with branch-and-price and column generation to solve huge integer programs [Barnhart *et al.* 1998], this issue is our future work and not within this paper. The crucial issue of schedule with two pipelines transfers and high-fusion-point oil is the scheduling of H-oil which requires enough L-oil in some charging tank to heat the pipeline2 and enough capacity in charging tanks to accept H-oil so as to continuously transport H-oil. So we can first create the H-oil scheduling model, if this model has optimal solution, we create the L-oil transfer model using the above solution to schedule the L-oil transfer.

## CONCLUSIONS

Due to the combinatorial characteristic and complicated scheduling essential demands of short-term crude oil operations scheduling in a refinery, it is an NP-hard problem. The problem must satisfy a series of constraints including resource constraints and process ones that arouse an extraordinary difficulty even to discover a feasible schedule. The problem presented in this paper is more academically challenging than the previous one-pipeline system because H-oil transfer needs high cost and it has to share tanks with L-oil. a new objective is introduced by the cost consideration, namely, to maximize the H-oil volume on the basis of any individual H-oil transfer setup. As far as the authors have known, this paper is the seminal one to formally academically research the two-pipeline systems scheduling problem using mathematical programming method. It uses mathematical programming method to obtain a feasible schedule which transports as much H-oil as possible and to minimize the setup cost of H-oil. This model is MINLP model, so it cannot be solved effectively by CPLEX which is one of the most popular and

efficient commercial software for mathematical programming and a simple two stage MILP-NLP procedure just like as done in [Mouret *et al.*, 2009] also cannot be used to solve this model because the nonlinear item contains binary decision variable which has effect on the assignment of operation. So we have to use the generic solver DICOPT to solve this model. We propose Algorithm 1 using DICOPT solver. It reveals that the system initial state has great effect on the H-oil amount that can be transferred, especially the flow rate of pipeline2. Therefore, before starting H-oil transfer we must design a plan of the initial conditions. The effectiveness of the proposed methodology has been testified through the case study results.

Considering optimization objectives else to satisfy diverse industrial demands is our future work, e.g., inventory cost minimization, discharging and charging loss minimization, crude oil transfer switch cost minimization, and minimizing charging switch cost in charging distillers and develop a software tool to implement the presented methodology. On the other hand, when our model is huge, how to solve it using commercial software combined with branch-and-price and column generation also is our important future issue.

## ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under grants 61273036.

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