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Quantum-Brownian Motion and Brownian Bridge Path Integral of a Free Particle

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Abstract: As we learned already, there is the quantum –Brownian motion generated by the quantum fluctuation in vacuum. According to the Langevin equation of fluctuation path [1], we have derived the new quantum-Brownian motion with variance $\overline{\Delta x^2} = 2(\frac{\hbar}{2m})\Delta\tau$; and the solution of this Langevin equation is a Brownian bridge. And then we have suggested the new theory of Brownian bridge path integral of a free particle, which gives the best description for wave particle dualism.

Keywords: Quantum-Brownian Motion; Brownian Bridge; Path Integral

INTRODUCTION

Why did a free particle have probability distribution? Is there trajectory of a free particle? We should go further into these questions. The independent –incremental process of production and annihilation of virtual particle pairs in vacuum fluctuation be called as quantum-Brownian motion, but the quantum-diffusional coefficient D_Q be not calculated by anyone. In this paper, we have derived $D_Q = (\frac{\hbar}{2m})$ as the starting point of our theory.

I. Quantum-Brownian Motions of Vacuum Fluctuation and Free particle

A.The Quantum-Brownian Motions of Vacuum Fluctuation

We known that $\{A_t^* \mid t \geq 0\}$ and $\{A_t \mid t \geq 0\}$ respectively represent production operator and annihilation operator of virtual particle pairs in vacuum [2], and

$$Q_t = A_t^* + A_t, t \geq 0 \quad , \tag{1a}$$

the quantum-Brownian motion is defined by

$$A_t^* + A_t = \int_0^t (\partial_t^* + \partial_t), \tag{1b}$$

where $(\partial_s^* + \partial_s)$ is called the quantum white noise w_s .

In this paper,first,we think the following functional equation [2]

$$(A_t^* + A_t)\phi = B_Q(t)\phi \tag{1c}$$

should be a operator equation,where φ is functional wave function.The operators $(A_t^* + A_t)$ and $B_Q(t)$ act on φ of two sides in equation (1c), which should be equal to the product of the fluctuation energy of virtual particle pairs and functional wave function φ .Therefore, we conclude that equation (1c)should be astochastic differential equation.

B. The Quantum-Brownian Motion of a Free particle

i). The mean square displacement of quantum-Brownian motion of a free particle

According to Einstein's views [3]: The success of statistical interpretation of wave mechanics means that the motion of the particle has the property of Brownian motion. And we have learned that the classical field ϕ at a given point behaves like a Brownian particle [4]. There is the white noise generated by quantum fluctuations, which leads to the Brownian motion of the classical field ϕ . For T and $\hbar$ both finite, there must be the thermal and quantum fluctuations, and the fluctuating path in the path integral obeys a "Langevin equation" [1]. From these views we will research into the quantum Brownian motion for a free particle. On the boundary between a particle and vacuum can generate the non-zero stochastic vacuum energy $B_Q(t)$, vacuum momentum $P_Q(t)$ and pressure, which are generated by the virtual particles in vacuum fluctuation.

We will further give a new Langevin equation

$$\frac{dx_t}{dt} - \upsilon(x(t),t) = (\frac{1}{c})\frac{B_Q(t)}{m}, \tag{1d}$$

Where $B_Q(t)$ is the independent-incremental process of the energy of virtual particle pairs in production and annihilation processes, which describes the quantum fluctuation of vacuum energy. Therefore, the quantum-Brownian motion $B_Q(t)$ is actually Gaussian-stochastic process of the energy change for virtual particle pairs. Equation (1d) can be rewritten as the following form[1]

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$$\frac{dx(t)}{dt} - \upsilon(x(t),t) = \frac{P_Q(t)}{m}, \tag{1e}$$

which is just that the fluctuating path in the path integral obeys a stochastic differential equation [1]

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} - \upsilon(x(t),t) = \frac{P(t)}{m}, \tag{1f}$$

or $$\frac{\mathrm{d}x(\tau)}{\mathrm{d}\tau} - \upsilon(x(\tau),\tau) = \frac{P(\tau)}{m}, \tag{1g}$$

which is analogous to the classical Langevin equation. The momentum $P(t)$ in equations(1e,1f,1g) plays the role of the noise with the correlation function [5]

$$< P(t)P(t^{'}) >= -im\hbar\delta(t-t^{'}), \tag{2}$$

which has the same correlation function as the white noise. Because $t=-i\tau$ and $\delta(ax)=a^{-1}\delta(x)$, thus Eq.2 becomes

$$\begin{aligned}\langle P(\tau)P(\tau')\rangle &= -im\hbar\delta[(-i\tau)-(-i\tau')] = -im\hbar\delta[(-i)(\tau-\tau')] \\ &= (-i)m\hbar(-i)^{-1}\delta(\tau-\tau') = m\hbar\delta(\tau-\tau')\end{aligned} \tag{3}$$

Equation (1g) has the solution as following form

$$x = x_0 + \upsilon\tau + \int_0^\tau \frac{P(s)}{m}\mathrm{d}s \tag{4}$$

Obviously the last term in Eq.4 shows that the positional fluctuation of a particle deviating from its classical path, and the mean-square-displacement of a particle can be calculated by using the following method

$$\begin{aligned}\overline{\{x(\tau)-x(0)\}^2} &= \int_0^\tau \mathrm{d}\tau_1\int_0^\tau \mathrm{d}\tau_2\overline{\upsilon(\tau_1)\upsilon(\tau_2)} = \int_0^\tau \mathrm{d}\tau_1\int_0^\tau \mathrm{d}\tau_2\overline{(\frac{P(\tau_1)}{m})(\frac{P(\tau_2)}{m})} \\ &= \int_0^\tau \mathrm{d}\tau_1\int_0^\tau \mathrm{d}\tau_2\{\frac{m\hbar\delta(\tau_1-\tau_2)}{m^2}\} = \int_0^\tau \mathrm{d}\tau_1\int_0^\tau \mathrm{d}\tau_2\{(\frac{\hbar}{m})\delta(\tau_1-\tau_2)\}\end{aligned} \tag{5}$$

Now we transform the integral variables into $\tau_1-\tau_2=\tau^{'}$ and $\tau_2(0<\tau_2<\tau-\tau^{'})$, which shows in Figure1, thus we have

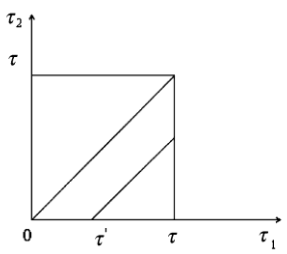

Figure 1. The integral region is equal to 2 times the size of the following triangle.

$$\begin{aligned}2\int_0^\tau d\tau^{'}\int_0^{\tau-\tau^{'}} d\tau_2(\frac{\hbar}{m})\delta(\tau^{'}) &= 2(\frac{\hbar}{m})(\tau-\tau^{'})\int_0^\tau \delta(\tau^{'})d\tau^{'} \\ &= 2\left(\frac{\hbar}{m}\right)\tau\int_0^\tau \delta(\tau')d\tau' - 2\left(\frac{\hbar}{m}\right)\int_0^\tau \tau'\delta(\tau')d\tau' \\ &= 2\left(\frac{\hbar}{2m}\right)\tau\end{aligned} \tag{6}$$

In Eq.6, we have used the properties of δ - function[6]:

$$\int_0^\tau \tau^{'}\delta(\tau^{'})d\tau^{'} = 0, \tag{7}$$

Since $\tau^{'}\delta(\tau^{'})=0$ and $\int_0^\tau \delta(\tau')d\tau' = 1$, inserting Eq.6 into Eq.5, we get

$$\overline{\{x(\tau)-x(0)\}^2} = 2\left(\frac{\hbar}{2m}\right)\tau. \tag{8}$$

Eq.8 is the mean square displacement of the quantum-Brownian motion for a free particle. Comparing with the Einstein's Brownian motion, the mean square displacement is

$$\overline{\{x(t)-x(0)\}^2} = 2Dt, \tag{9}$$

where $D=\frac{kT}{\kappa}$ is the diffusion coefficient for thermal fluctuation. Thus, we can conclude that $(\frac{\hbar}{2m}) \equiv D_Q$ in Eq.(8) should be interpreted to be the diffusion coefficient for quantum fluctuation. Comparing the diffusion coefficient $D=\frac{kT}{\kappa}$ for thermal fluctuation and the diffusion coefficient $D_Q \equiv (\frac{\hbar}{2m})$ for quantum fluctuation, they have extremely interesting symmetry.

Since the fluctuation path in the path integral obeys the Langevin Eq.1, thus we can think a quantum-mechanical particle behaves like a "Brownian particle". And the momentum $P(t)$ in Eq.1 plays the role of the Gaussian white noise, which leads to the "Brownian motion" of a quantum-mechanical particle.

ii). The solution of quantum-Brownian motion equation-Brownian bridge

We see in Eq.(8) that the mean-square-root deviation σ_Q of the quantum-Brownian motion of a free particle should be

$$\sigma_Q = \{2(\frac{\hbar}{2m})\tau\}^{\frac{1}{2}}. \tag{10}$$

Because $P(\tau)$ in (1b) plays the role of the noise, we may write $P(\tau)=w(\tau)=\frac{dB(\tau)}{d(\tau)}$, (11a)

Thus, we can rewrite the solution (4) of Langevin equation (1g) as the follwing

$$x = x_0 + \upsilon\tau + \int_0^\tau \frac{P(s)}{m}ds = x_0 + \upsilon\tau + x_\tau^{q',q''}, \tag{11b}$$

where $x_\tau^{q',q''}$ should be considered as Brownian bridge, which should be the fluctuation Brownian paths deviating from classical path $(x_0+\upsilon\tau)$ between two points $q'(0)$ and $q''(\tau)$.

BROWNIAN BRIDGE PATH INTEGRAL OF A FREE PARTICLE

We think a free particle moving in Brownian bridge is the conditional stochastic process. We have calculated the conditional expectation for the probabilty amplitudes of a free particle moving along respective possible Brownian bridge paths $q(t)$ to the probability amplitude of boundary interval $(q''-q')$. Thus we have yet obtained the amplitude distribution of the modulated plane wave, and its modulation factor is also Gaussian function. We have given the best description for wave particle dualism and some new interpretations on wave function.

A. Rewrite Feynman's Path Integral by using Brownian Bridge

Any possible path $q(t)$ of a free particle moving in Brownian bridge must pass through two boundary points $(0,q')$ and (t_0,q'') as shown in figure2.

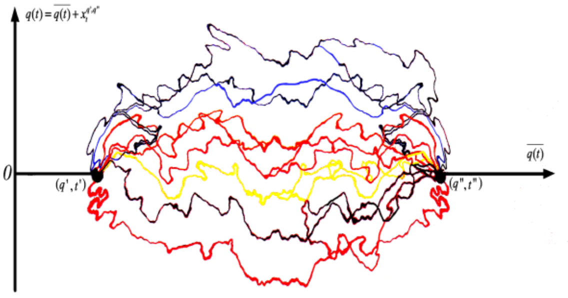

Figure 2. Where we let $q(t)$ be a lot of Brownian bridge paths, $\overline{q(t)}$ be average classical trajectory. Let (q',t') and (q'',t'') be the two possible boundary points on $\overline{q(t)}$, and x_t be path fluctuations deviating from classical trajectory $\overline{q(t)}$.

Let $\overline{q(t)}$ be the classical trajectory of a free particle, and we may give the new sense for formula

$$q(t)=\overline{q(t)}+x_t^{q',q''}, \qquad (12)$$

where we let $x_t^{q',q''}$ be Brownian bridge, which may be difined by the following formula [6]

$$x_t^{q',q''}=B_t+q'+\frac{t}{t_0}(q''-q'-B_{t_0}). \qquad (13)$$

Let $x_0^{q',q''}=q', x_{t_0}^{q',q''}=q''$, and $x_t^{q',q''}$ may be written as x_t. Put t $0<t_1<t_2<\cdots<t_n\le t_0=t_{n+1}$, at $t'=0$ and $t''=t_0, q(t)$ must pass through two boundary points $(0,q')$ and (t_0,q''). Now, we let $q(t)$ be any Brownian bridge trajectory of a free particle, x_t be the path fluctuations of a free particle deviating from the average classical trajectory $\overline{q(t)}$.

We will rewrite Feynamn's path integral by using Brownian bridge [1],[6],[7]. As shown in figure 2, a free particle starting from the initial point $(0,q')$ can arrive the given end point (t_0,q'') along all the possible paths $q(t)$ of Brownian bridge. There are the probability amplitudes $e^{(\frac{i}{\hbar})S[q(t)]}$ and phase change along respective bridge paths $q(t)$. There is only sample probability amplitude for single choosing path, but no probability [8],[9]. And the probalility density of a free particle moving in Brownian bridge should obtain from its transition probability amplitude in Brownian bridge path integral.

B.Brownian Bridge Path Integral

The action along any Brownian bridge path $q(t)$ between $(0,q')$ and (t_0,q'') should be [7]

$$S[q(t)]=S[\overline{q(t)}]+S[x_t]. \qquad (14)$$

The total probablity amplitude of a free particle starting from the initial point $(0,q')$ to end point (t_0,q'') should obtain by using the coherent superposition of the respective probability amplitudes along all the possible paths of Bronian bridge. The total probability amplitude is the transition probability amplitude of a free particle moving in Brownian bridge, thus we have

$$<q''t''|q't'>=\int g_{q''|q'}[q(t)]\mathfrak{D}q(t)e^{\frac{i}{\hbar}S[q(t)]}, \qquad (15)$$

where $g_{q''|q'}[q(t)]$ is the superposition coefficient, which should be the conditional Gaussian function. The probability amplitudes along respective bridge paths $q(t)$ are

$$\begin{aligned} e^{\frac{i}{\hbar}S[q(t)]} &= e^{\frac{i}{\hbar}S[\overline{q(t)}+x_t]} = e^{\frac{i}{\hbar}\int_0^{t_0}\frac{1}{2}m[\dot{\overline{q(t)}}+\dot{x}_t]^2dt} \\ &= e^{\frac{i}{\hbar}\int_0^{t_0}\frac{1}{2}m\dot{\overline{q(t)}}^2dt}\, e^{\frac{i}{\hbar}m\dot{\overline{q(t)}}\int_0^{t_0}dx_t}\, e^{\frac{i}{\hbar}\int_0^{t_0}\frac{1}{2}m\dot{x}_t dx_t}. \end{aligned} \qquad (16)$$

Differentiating(13), we have

$$\begin{cases} dx_t=dB_t+\dfrac{(q''-q'-B_{t_0})}{t_0}dt, \\ dx_t^2=(dB_t)^2+\dfrac{2(q''-q'-B_{t_0})}{t_0}dtdB_t+\left(\dfrac{q''-q'-B_{t_0}}{t_0}\right)^2dtdt. \end{cases} \qquad (17)$$

Because $dtdB_t=0, dtdt=0$, and

$$(dx_t)^2=(dB_t)^2=2D_Qdt, \qquad (18)$$

where D_Q is the quantum diffusion coefficient, we have proven that the variance $\overline{\delta x_t^2}$ of positional fluctuation of a free particle in quantum-Brownian motion is

$$\overline{\{x(\tau)-x(0)\}^2}=2\left(\frac{\hbar}{2m}\right)\tau=2D_Q\tau, \qquad (19)$$

where $\tau=it$. Thus, we may write

$$e^{\frac{im}{2\hbar}\int_0^{t_0}\frac{(dx_t)^2}{dt}}=e^{\frac{im}{2\hbar}\int_0^{t_0}\frac{(dB_t)^2}{dt}}=e^{\frac{im}{2\hbar}\lim\limits_{\Delta t\to 0}\sum_{j=1}^{n+1}2D_Q\frac{\Delta t_j}{\Delta t}}=e^{\frac{im}{2\hbar}(n+1)2D_Q}$$

in formula (16). Therefore, by using the view point of Brownian bridge, the Brownian bridge path integral (15) of a free particle can be re-written as

$$<q''t''|q't'>=\int_{=e^{\frac{im}{2\hbar}\int_0^{t_0}\dot{\overline{q(t)}}^2dt}e^{\frac{im(n+1)2D_Q}{2\hbar}}}^{g_{q''|q'}[q(t)]\mathfrak{D}q(t)e^{\frac{i}{\hbar}S[q(t)]}}\left\{\int g_{q''|q'}[x_t]e^{\frac{im\dot{\overline{q(t)}}}{\hbar}\int_0^{t_0}dx_t}\mathfrak{D}x_t\right\}, \qquad (20)$$

which shows that the integration variable has changed from Brownian bridge path $q(t)$ to the path fluctuation x_t deviating from average classical trajectory $\overline{q(t)}$.

In formula (20), we rewrite

$$e^{\frac{im\dot{\overline{q(t)}}}{\hbar}\int_0^{t_0}dx_t}=e^{\frac{im\dot{\overline{q(t)}}}{\hbar}\lim\sum_{j=1}^{n+1}(x_j-x_{j-1})}=e^{\frac{im\dot{\overline{q(t)}}}{\hbar}\lim\sum_{j=1}^{n+1}\Delta x_j}, \qquad (21)$$

we see in (21) that the path fluctuation x_t can be considered as the sum of many independent increments Δx_j. Inserting (21) into (20), we have

$$<q"t"|q't'>=e^{\frac{im}{2\hbar}\int_0^{t_0}\overset{\bullet}{q(t)}^2}e^{\frac{im}{2\hbar}(n+1)2D_Q}\cdot$$
$$\left\{\int g_{q"|q'}[\Delta x_1,\Delta x_2,\cdots\Delta x_{n+1}]e^{\frac{im\overset{\bullet}{q(t)}}{\hbar}\lim\sum_{j=1}^{n+1}\Delta x_j}\prod_{j=1}^{n+1}d(\Delta x_j)\right\},\quad (22)$$

where the conditional Gaussian function $g_{q"|q'}[x_t]$ should be written as the follwing form [5],[10]

$$g_{q"|q'}[\Delta x_1,\Delta x_2,\cdots\Delta x_{n+1}]=\prod_{j=1}^{n+1}\left\{\frac{e^{-\Delta x_j^2/4\sigma_j^2}}{(\sqrt{2\pi\sigma_j^2})^{\frac{1}{2}}}\right\}\left[\frac{e^{-(q"-q')^2/4D_Q(t"-t')}}{(\sqrt{2\pi 2D_Q(t"-t')^2})^{\frac{1}{2}}}\right]^{-1}.\quad (23)$$

The mathematical form of Brownian bridge path integral (20) is analogous to the calculation of the conditional expectation for the probability amplitudes along respective Brownian brigde paths $q(t)$ to the probability amplitude of boundary interval $(q"-q')$[10].

We integrate respectively to each independent increment $|\Delta x_j|$ of path fluctuation in (22), by using Strotonovich stochastic integral, which has usual integral method, we have

$$I_{\Delta x_j}=\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi\sigma_j^2}}\right)^{\frac{1}{2}}e^{\frac{-|\Delta x_j|^2}{4\sigma_j^2}}e^{\frac{im\overset{\bullet}{q(t)}}{\hbar}|\Delta x_j|}d|\Delta x_j|=\left(\frac{1}{\sqrt{2\pi\sigma_j^2}}\right)^{\frac{1}{2}}\left\{(2\sigma_j)e^{-\lambda^2\sigma_j^2}\int_{-\infty}^{\infty}e^{-(u-i\sigma_j\lambda)^2}du\right\},$$

(24)

where $u\equiv\frac{|\Delta x_j|}{2\sigma_j},\lambda\equiv\frac{m\overset{\bullet}{q(t)}}{\hbar},du\equiv\frac{d|\Delta x_j|}{2\sigma_j}$, thus we can write

$$I=\int_{-\infty}^{\infty}e^{-(u-i\sigma_j\lambda)^2}du=\int_{-\infty}^{\infty}e^{-\xi^2}d\xi,\quad (25)$$

where $\xi\equiv u-i\sigma_j\lambda, du=d\xi,$ thus we have

$$I^2=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-(\xi^2+\eta^2)}d\xi d\eta,\quad (26)$$

By using polar coodinates, we obtain

$$I^2=\int_0^{\infty}\int_0^{\pi}e^{-r^2}rdrd\theta=\pi\int_0^{\infty}e^{-r^2}dr^2=\pi,\quad (27)$$

therefore

$$I_{\Delta x_j}=\left(\frac{1}{\sqrt{2\pi\sigma_j}}\right)^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})e^{-\lambda^2\sigma_j^2}.\quad (28)$$

Inserting (28) into (22), we obtain

$$<q"t"|q't'>=$$
$$e^{\frac{im}{2\hbar}\int_0^{t_0}\overset{\bullet}{q(t)}^2 dt}e^{\frac{im}{2\hbar}(n+1)2D_Q}\cdot\left[\frac{e^{-\frac{(q"-q')^2}{4D_Q(t"-t')}}}{(\sqrt{4\pi D_Q(t"-t')})^{\frac{1}{2}}}\right]^{-1}\prod_{j=1}^{n+1}\left(\frac{1}{\sqrt{2\pi\sigma_j^2}}\right)^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})e^{-\lambda^2\sigma_j^2}.$$

(29)

According to the formula (8) of quantum diffusion coefficient

$\overline{(\Delta x_j)^2}=\sigma_j^2=2D_Q\Delta\tau_j=2\left(\frac{\hbar}{2m}\right)i\Delta t_j$, and $\overline{x_t^2}=2\left(\frac{\hbar}{2m}\right)it,$

We can rewrite (29) as the following form

$$<q"t"|q't'>=$$
$$e^{\frac{im}{2\hbar}\int_0^{t_0}\overset{\bullet}{q(t)}^2 dt}e^{\frac{im}{\hbar}(n+1)D_Q}\cdot\left[\frac{e^{\frac{-(q"-q')^2}{4D_Q(t"-t')}}}{(\sqrt{4\pi D_Q(t"-t')})^{\frac{1}{2}}}\right]^{-1}\prod_{j=1}^{n+1}\left(\frac{1}{\sqrt{2\pi\sigma_j^2}}\right)^{\frac{1}{2}}(2\sigma_j\sqrt{\pi})$$

$$\left\{e^{-\left(\frac{m^2\overset{\bullet}{q(t)}^2}{\hbar^2}\right)\left[2\left(\frac{\hbar}{2m}\right)i\Delta t_j\right]}\right\}=$$

$$e^{\frac{im}{2\hbar}\int_0^{t_0}\overset{\bullet}{q(t)}^2 dt}e^{\frac{im}{\hbar}(n+1)D_Q}\left[\frac{e^{\frac{-(q"-q')^2}{4D_Q(t"-t')}}}{(\sqrt{4\pi D_Q(t"-t')})^{\frac{1}{2}}}\right]^{-1}\left(\frac{1}{\sqrt{2\pi\sigma_{x_t}^2}}\right)^{\frac{1}{2}}(2\sqrt{\pi}\sigma_{x_t})\left\{e^{\frac{-2i}{\hbar}\left(\frac{m\overset{\bullet}{q(t)}^2}{2}\right)t}\right\}$$

$$=\left(\frac{1}{\sqrt{2\pi\sigma_{x_t}^2}}\right)^{\frac{1}{2}}(2\sqrt{\pi}\sigma_{x_t}^2)e^{\frac{im}{\hbar}(n+1)D_Q}\left[\frac{e^{\frac{-(q"-q')^2}{4D_Q(t"-t')}}}{(\sqrt{4\pi D_Q(t"-t')})^{\frac{1}{2}}}\right]^{-1}e^{\frac{im}{2\hbar}\int_0^{t_0}\overset{\bullet}{q(t)}^2 dt-\frac{i}{\hbar}2Et},$$

(30)

where $\sigma_{x_t}^2$ is the variance of path fluctuation x_t of a free particle. Take note of

(31)

$$e^{-\frac{i}{\hbar}Et}=e^{-\frac{1}{\hbar}E\tau}=e^{-\frac{1}{\hbar}\left[\frac{1}{2}m\left(\frac{\overline{q(\tau)}}{\tau}\right)^2\tau\right]}$$
$$=e^{-\frac{1}{\hbar}\left[\frac{1}{2}m\frac{\overline{q(\tau)}^2}{\tau}\right]\left(\frac{\frac{\hbar}{m}}{\frac{\hbar}{m}}\right)}=e^{-\frac{\overline{q(\tau)}^2}{2\left(\frac{\hbar}{m}\right)\tau}}=e^{\frac{-\overline{q(\tau)}^2}{2\sigma_\tau^2}}.$$

Because the linear operation of Gaussian processes should be also Gaussian process [5] ,thus the probability amplitudes of a free particle moving along paths $q(t)$, x_t and $\overline{q(t)}$ in Brownian bridge should have the same Gaussian distribution. Therefore, we may rewrite (30) as the following form

$$<q"\tau"|q'\tau'>=$$
$$(2\sqrt{\pi}\sigma_\tau^2)e^{\frac{im}{\hbar}(n+1)D_Q}\left[\frac{e^{\frac{-(q"-q')^2}{4D_Q(\tau"-\tau')}}}{(\sqrt{4\pi D_Q(\tau"-\tau')})^{\frac{1}{2}}}\right]^{-1}\left(\frac{1}{\sqrt{2\pi\sigma_\tau^2}}\right)^{\frac{1}{2}}e^{\frac{-\overline{q(\tau)}^2}{2\sigma_\tau^2}}\left\{e^{\frac{i}{2\hbar}\overline{p}\overline{q(\tau)}-\frac{i}{\hbar}\overline{E}\tau}\right\}$$

（32)

where σ_τ^2 is the positional variance of a free particle moving in average classical trajectory $\overline{q(\tau)}$.

C.Transmiting Amplitude Wave Function in Brownian Bridge

According to Huygens-Fresnel principle of transmiting amplitude wave $\psi(x,t)$[7]

$$\psi(x_2,t_2)=\int_{-\infty}^{\infty}<x_2,t_2|x_1,t_1>\psi(x_1,t_1)dx_1,\quad (33a)$$

we may rewrite formula (32), let $q'(\tau')$ and $q"(\tau")$ are the variable boundary points, and $\overline{q(\tau)}$ is variable average classical trajectory in Brownian bridge. Thus, we have

$$\psi\,(\overline{q(\tau)}) = \int_{-\infty}^{\infty} < \overline{q(\tau)}\,|\,(q''-q') > \frac{1}{(\sqrt{4\pi D_Q(\tau''-\tau')})^{\frac{1}{2}}} e^{\frac{-(q''-q')^2}{4D_Q(\tau''-\tau')}} d(q''-q')$$

$$= (2\sqrt{\pi}\sigma_\tau^2) e^{\frac{im}{\hbar}(n+1)D_Q} \left(\frac{1}{\sqrt{2\pi\sigma_\tau^2}}\right)^{\frac{1}{2}} e^{\frac{-\overline{q(\tau)}^2}{2\sigma_\tau^2}} \left\{ e^{\frac{i}{2\hbar}\bar{p}\overline{q(\tau)} - \frac{i}{\hbar}\bar{E}\tau} \right\} \int_{|(q''-q')|}^{|(q''-q')|+\sigma_\tau} d(q''-q')$$

$$\left[\frac{e^{\frac{-(q''-q')^2}{4D_Q(\tau''-\tau')}}}{(\sqrt{4\pi D_Q(\tau''-\tau')})^{\frac{1}{2}}}\right]^{-1} \left[\frac{e^{\frac{-(q''-q')^2}{4D_Q(\tau''-\tau')}}}{(\sqrt{4\pi D_Q(\tau''-\tau')})^{\frac{1}{2}}}\right], \qquad (33b)$$

which is multiplied by marginal probability amplitude on two sides of formula (32), and integrating for the following boundary condition [5]: the probability density

$$f\left\{\left[|(q''-q')|+\sigma_\tau\right] = \pm\infty, \tau\right\} = 0, \qquad (33c)$$

which shows that the fluctuating boundary distance $\left[|(q''-q')|+\sigma_\tau\right]$ keep finite values.

We see in (33b) that the integral result on left side should be the amplitue wave function $\psi(\overline{q(\tau)})$, and integral result on right side is σ_τ. Thus, formula (33b) becomes

$$\psi(\overline{q(\tau)}) = (2\sqrt{\pi}\sigma_\tau^3) e^{\frac{im}{\hbar}(n+1)D_Q} \left(\frac{1}{\sqrt{2\pi\sigma_\tau^2}}\right)^{\frac{1}{2}} e^{\frac{-\overline{q(\tau)}^2}{2\sigma_\tau^2}} \left\{ e^{\frac{i}{2\hbar}\bar{p}\overline{q(\tau)} - \frac{i}{\hbar}\bar{E}\tau} \right\}, \qquad (34)$$

which is just the modulated plane wave, and the amplitude modulation foctor is also Gaussian function, the peak is at $\overline{q(\tau)} = 0$, when $\sigma_\tau \to 0, \psi(\overline{q(\tau)})$ is $\delta-$functional wave packet [8], its width is $2\sigma_\tau$, which should diffuse with time τ as Gaussian wave packet, as shown in figure3.

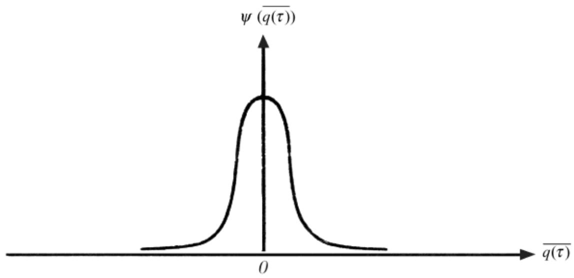

Figure 3. The peak of $\psi(\overline{q(\tau)})$ is at $\overline{q(\tau)}$ =0, $\psi(\overline{q(\tau)})$ is $\delta-$functional wave packet, when $\sigma_\tau \to 0$.

CONCLUSION

As mentioned above, we have rewritten Feynman's path integral into Brownian bridge pach integral, which is the best description for wave particle dualism. First, we can clearly desribe the stochastic motions of a free particle on respective possible sample bridge paths $q(\tau)$, path fluctuations x_τ and average classical trajectory $\overline{q(\tau)}$, and we think these stochastic motions of a free particle in Brownian bridge should be its quantum Brownian motion. Next,we think respective probability amplitudes $e^{\frac{i}{\hbar}S[q(\tau)]}$ of a free particle moving along respective Brownian bridge paths $q(\tau)$ are the periodic sample functions of stochastic process in Brownian bridge. And these sample bridge paths $q(\tau)$ and correponding sample amplitude functions in stochastic Brownian bridge process should be simultaneous [5], thus, we can take the coherent superposttion to calculate the conditional expectation in Brownian brigde path integral; and we can explain: Why did a free particle have the non-locatized connection and entangled state? It is just due to that these different sample bridge paths $q(\tau)$ and corresponding sample probability amplitudes of a free particle are simultaneous [5] in stochastic Brownian bridge process.

Comparing schrödinger equation

$$\frac{d}{dt}\varphi = \varepsilon\frac{d^2}{dx^2}, (\varepsilon = \frac{ih}{4\pi m}) \qquad (35)$$

and diffusion equation

$$\frac{d}{dt}\omega = D\frac{d^2}{dx^2}\omega\ , \qquad (36)$$

Fürth and L.F.Favella had proven that equations (35)and(36)are analogous[11].

Now, we can strictly prove that schrödinger equation is actually Fokker-Planck equation with quantum-diffusional coefficient. We rewrite schrödinger equation for a free particle as the following form

$$i\hbar\frac{\partial}{\partial t}\varphi(x,t) = -(\frac{\hbar^2}{2m})\frac{\partial^2}{\partial x^2}\varphi(x,t)\,, \qquad (37a)$$

which may rewrite as

$$\frac{\partial}{\partial t}\varphi(x,t) = (\frac{i\hbar}{2m})\frac{\partial^2}{\partial x^2}\varphi(x,t)\,, \qquad (37b)$$

comparing equations (11)and(13b),we have

$$\varepsilon = iD_Q\,, \qquad (37c)$$

where D_Q is just the quantum-diffusional coeffivient in quantum-Brownian motion for a free particle.Rewriting $(37b)$ as following form

$$\frac{\partial}{\partial\tau}\varphi(x,\tau) = (\frac{\hbar}{2m})\frac{\partial^2}{\partial x^2}\varphi(x,\tau) = D_Q\frac{\partial^2}{\partial x^2}\varphi(x,\tau)\,, \qquad (38)$$

which is just Fokker-Planck equation with quantum-diffusional coefficient D_Q,and its solution should be the transfer probability amplitude of the quantum-Brownian motion for a free particle.Thus,we can conclude that schrödinger eguation is actually Fokker-planck equation with quantum-diffusional coefficient D_Q.

REFERENCES

[1] H. Kleinert, 1995,Path Integral in Quantum Mechanics, Statistics and Polymer Physics. New York: World Scientific Publishing Company.

[2] Huang Zhiyuan,Quantum White Noise Analysis(in chinese).

[3] L. de Broglie, 1960,Non-linear wave mechanics (A causal interpretation) ,Elsevier Publishing Co.

[4] A. Linde, 1994,From the Big Bang Theory to the Theory of a Stationary Universe, Phys. Rev. D. 49.

[5] T. T. Soong, Random Differential Equation in Science and Engineering. New York.

[6]Gong Guanglu,1987,Introduction to Stochastic Differential Equations (in Chinese), Beijing university Publishing Company,Brownian Bridge, pp 312-323

[7] R. P. Feynman and A. R. Hibbs,1965,Quantum Mechanics and Path Integrals. New York: Mcgraw-Hilll Book Company.

[8] C. Cohen-Tannoudji, 1977,Quantum Mechanics, Vol.1. A Wiley-Interscience Publication, New York. pp262-263: "Young's double-slit experiment, must one sum the amplitudes or the probabilities?" "A photon simultaneously come out from double-slit, and the probability detecting it at some point should be the square of sum of the simultaneous amplitudes along two possible paths."

[9] R. Penrose, 1989,The Emperor's New Mind, Oxford University Press,Chapter 6: "Double-slit experiment, a photon simultaneously pass through tow slits, and a photon simultaneously take the peak values of probability amplitudes at two places, when it have came out from two slits."

[10] E. Parzen, Stochastic processes, 1967,Chapter 2: Conditional Probability and Conditional Expectation.

[11]L.F.Favella, "Brownian motion and quantum mechanics",Annales de L'Institut Henri Poincare' 7.pp77-94.