

H_∞ Model Reduction for Discrete-time 2D Markovian Jump System with State Delays in Roesser Model

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Abstract: This paper is considered with the problem of H_∞ model reduction for a class of discrete-time 2D Markovian jump systems with state delays described by the Roesser model. Since these obtained conditions are not expressed as strict LMIs, the cone complementarily linearization (CCL) method is exploited to cast them into nonlinear minimization problems subject to LMI constraints. A numerical example is given to illustrate the design procedures.

Introduction

With the development of modern industry and economy, more and more multivariable systems and multidimensional signal need to be handled. Such as multi-dimensional digital image processing, multivariable network realization, meteorological satellite image analysis, which are mostly appear as 2D discrete system model. For these profound engineering backgrounds, in recent years, 2D discrete systems have received much attention, and many important results are available in the literatures [1, 2].

On a different direction, a considerable research effort has been recently devoted to the analysis of a kind of hybrid systems -- Markovian jump system whose structures are subject to random abrupt changes may due to component or interconnections failures, sudden environment changes, change of the operating point of a linearized model of a nonlinear, and so on. The application of Markovian jump systems can be found in many physical systems, such as manufacturing systems, target tracking, and power system [3-4]. And some problems of stability, controller design and filtering related to these systems also have been extensively studied by numerous scholars, see for instance [5, 6], and the references therein. Since delay usually occur in many physical and engineering systems and causes instability and poor performance of systems, time-delay systems have been studied extensively on the subject of control and model reduction over the years. For example, in [7] Wang *et al.* addressed the model approximation for discrete-time Markovian jump systems with mode-dependent time delays. However, the aforementioned results are just concerned with one-dimensional systems, and to the best of the authors' knowledge, few effort has been made toward investigating the problems arising in 2D jump systems.

In this paper, we extends the sufficient conditions in terms of LMIs plus matrix inverse constraints are derived for the existence of a solution to the reduced-order model problems. Since these obtained conditions are not expressed as strict LMIs, the CCL method is exploited to cast them into nonlinear minimization problems subject to LMI constraints, which can be readily solved by standard numerical soft ware. A numerical example is given to illustrate the design procedures.

Problem formulation

Consider a n th-order 2D discrete delays system with Markovian jump parameters described by the Roesser model:

$$\begin{aligned} (\Sigma): \quad \begin{bmatrix} \mathbf{x}^h(i+1, j) \\ \mathbf{x}^v(i, j+1) \end{bmatrix} &= \mathbf{A}(r_{i,j}) \begin{bmatrix} \mathbf{x}^h(i, j) \\ \mathbf{x}^v(i, j) \end{bmatrix} + \mathbf{A}_{d1}(r_{i,j}) \mathbf{x}^h(i-d_1, j) + \mathbf{A}_{d2}(r_{i,j}) \mathbf{x}^v(i, j-d_2) + \mathbf{B}(r_{i,j}) \mathbf{w}(i, j) \\ \mathbf{z}(i, j) &= \mathbf{C}(r_{i,j}) \begin{bmatrix} \mathbf{x}^h(i, j) \\ \mathbf{x}^v(i, j) \end{bmatrix} + \mathbf{C}_{d1}(r_{i,j}) \mathbf{x}^h(i-d_1, j) + \mathbf{C}_{d2}(r_{i,j}) \mathbf{x}^v(i, j-d_2) + \mathbf{D}(r_{i,j}) \mathbf{w}(i, j) \end{aligned} \quad (1)$$

where $n = n_1 + n_2$, $\mathbf{x}^h(i, j) \in R^{n_1}$, $\mathbf{x}^v(i, j) \in R^{n_2}$ represent the horizontal and vertical states respectively; $\mathbf{w}(i, j) \in R^m$ is the disturbance input which is a square-integrable and norm bounded stochastic vector function over $L_2\{[0, \infty), [0, \infty)\}$; $\mathbf{z}(i, j) \in R^m$ is the controlled output; d_1 and d_2 are constant positive integers representing delays along horizontal direction and vertical direction, respectively. $\mathbf{A}(r_{i,j}) \in R^{n \times n}$, $\mathbf{A}_{d1}(r_{i,j}) \in R^{n \times n_1}$, $\mathbf{A}_{d2}(r_{i,j}) \in R^{n \times n_2}$, $\mathbf{B}(r_{i,j}) \in R^{n \times m}$, $\mathbf{C}(r_{i,j}) \in R^{l \times n}$, $\mathbf{C}_{d1}(r_{i,j}) \in R^{l \times n_1}$, $\mathbf{C}_{d2}(r_{i,j}) \in R^{l \times n_2}$, $\mathbf{D}(r_{i,j}) \in R^{l \times m}$ are matrix functions of the time-varying parameter $r_{i,j}$. The parameter $r_{i,j}$ takes values in a finite set $S = \{1, 2, \dots, s\}$, with transition probabilities

$$\Pr\{r_{i+1,j} = n \mid r_{i,j} = m\} = \Pr\{r_{i,j+1} = n \mid r_{i,j} = m\} = p_{mn}$$

where $p_{mn} \geq 0$, and for any $m \in S$, $\sum_{n=1}^s p_{mn} = 1$.

In this paper, our purpose is to find a mean-square asymptotically stable \tilde{n} th-order 2D jump system

$$\begin{aligned} \begin{bmatrix} \tilde{\mathbf{x}}^h(i+1, j) \\ \tilde{\mathbf{x}}^v(i, j+1) \end{bmatrix} &= \tilde{\mathbf{A}}_m \begin{bmatrix} \tilde{\mathbf{x}}^h(i, j) \\ \tilde{\mathbf{x}}^v(i, j) \end{bmatrix} + \tilde{\mathbf{A}}_{d1m} \tilde{\mathbf{x}}^h(i-d_1, j) + \tilde{\mathbf{A}}_{d2m} \tilde{\mathbf{x}}^v(i, j-d_2) + \tilde{\mathbf{B}}_m \mathbf{w}(i, j) \\ \tilde{\mathbf{z}}(i, j) &= \tilde{\mathbf{C}}_m \begin{bmatrix} \tilde{\mathbf{x}}^h(i, j) \\ \tilde{\mathbf{x}}^v(i, j) \end{bmatrix} + \tilde{\mathbf{C}}_{d1m} \tilde{\mathbf{x}}^h(i-d_1, j) + \tilde{\mathbf{C}}_{d2m} \tilde{\mathbf{x}}^v(i, j-d_2) + \tilde{\mathbf{D}}_m \mathbf{w}(i, j) \end{aligned} \quad (2)$$

$$\begin{aligned} \bar{\mathbf{x}}'(i, j) &= \bar{\mathbf{A}}(r_{i,j}) \bar{\mathbf{x}}(i, j) + \bar{\mathbf{A}}_{d1}(r_{i,j}) \bar{\mathbf{x}}_{d_1}^h(i, j) + \bar{\mathbf{A}}_{d2}(r_{i,j}) \bar{\mathbf{x}}_{d_2}^v(i, j) + \bar{\mathbf{B}}(r_{i,j}) \mathbf{w}(i, j) \\ \bar{\mathbf{z}}(i, j) &= \bar{\mathbf{C}}(r_{i,j}) \bar{\mathbf{x}}(i, j) + \bar{\mathbf{C}}_{d1}(r_{i,j}) \bar{\mathbf{x}}_{d_1}^h(i, j) + \bar{\mathbf{C}}_{d2}(r_{i,j}) \bar{\mathbf{x}}_{d_2}^v(i, j) + \bar{\mathbf{D}}(r_{i,j}) \mathbf{w}(i, j) \end{aligned} \quad (3)$$

where $\bar{\mathbf{x}}'(i, j) = \begin{bmatrix} \mathbf{x}'(i, j) \\ \tilde{\mathbf{x}}'(i, j) \end{bmatrix}$, $\bar{\mathbf{x}}(i, j) = \begin{bmatrix} \mathbf{x}(i, j) \\ \tilde{\mathbf{x}}(i, j) \end{bmatrix}$, $\bar{\mathbf{z}}(i, j) = \mathbf{z}(i, j) - \tilde{\mathbf{z}}(i, j)$, $\bar{\mathbf{x}}_{d_1}^h(i, j) = \begin{bmatrix} \mathbf{x}^h(i-d_1, j) \\ \tilde{\mathbf{x}}^h(i-d_1, j) \end{bmatrix}$,

$\bar{\mathbf{x}}_{d_2}^v(i, j) = \begin{bmatrix} \mathbf{x}^v(i, j-d_2) \\ \tilde{\mathbf{x}}^v(i, j-d_2) \end{bmatrix}$, $\mathbf{x}'(i, j) = \begin{bmatrix} \mathbf{x}^h(i+1, j) \\ \mathbf{x}^v(i, j+1) \end{bmatrix}$, $\tilde{\mathbf{x}}'(i, j) = \begin{bmatrix} \tilde{\mathbf{x}}^h(i+1, j) \\ \tilde{\mathbf{x}}^v(i, j+1) \end{bmatrix}$, $\mathbf{x}(i, j) = \begin{bmatrix} \mathbf{x}^h(i, j) \\ \mathbf{x}^v(i, j) \end{bmatrix}$,

$$\tilde{\mathbf{x}}(i, j) = \begin{bmatrix} \tilde{\mathbf{x}}^h(i, j) \\ \tilde{\mathbf{x}}^v(i, j) \end{bmatrix}, \quad \bar{\mathbf{A}}_m = \begin{bmatrix} \mathbf{A}_m & 0 \\ 0 & \tilde{\mathbf{A}}_m \end{bmatrix}, \quad \bar{\mathbf{A}}_{d1m} = \begin{bmatrix} \mathbf{A}_{d1m} & 0 \\ 0 & \tilde{\mathbf{A}}_{d1m} \end{bmatrix}, \quad \bar{\mathbf{B}}_m = \begin{bmatrix} \mathbf{B}_m \\ \tilde{\mathbf{B}}_m \end{bmatrix}, \quad \bar{\mathbf{A}}_{d2m} = \begin{bmatrix} \mathbf{A}_{d2m} & 0 \\ 0 & \tilde{\mathbf{A}}_{d2m} \end{bmatrix},$$

$$\bar{\mathbf{C}}_m = \begin{bmatrix} \mathbf{C}_m & -\tilde{\mathbf{C}}_m \end{bmatrix}, \quad \bar{\mathbf{C}}_{d1m} = \begin{bmatrix} \mathbf{C}_{d1m} & -\tilde{\mathbf{C}}_{d1m} \end{bmatrix}, \quad \bar{\mathbf{C}}_{d2m} = \begin{bmatrix} \mathbf{C}_{d2m} & -\tilde{\mathbf{C}}_{d2m} \end{bmatrix}, \quad \bar{\mathbf{D}}_m = \mathbf{D}_m - \tilde{\mathbf{D}}_m.$$

is mean-square asymptotically stable and has H_∞ performance.

In this paper, we take the following assumptions.

Definition 1: The error 2D jump system (3) with $\mathbf{w}_{i,j} = 0$ is said to be mean-square asymptotically stable if

$$\lim_{i+j \rightarrow \infty} \mathbb{E} \left\{ \|\bar{\mathbf{x}}_{i,j}\|^2 \right\} = 0$$

for every boundary condition $(\bar{\mathbf{X}}_0, \mathbf{R}_0)$ satisfying Assumption 1.

Definition 2: For a given scalar $\gamma > 0$, the error 2D jump system (3) is said to be mean-square asymptotically stable with an H_∞ disturbance attenuation level γ if it is mean-square asymptotically stable and under zero boundary condition $\bar{\mathbf{X}}_0 = 0$, $\|\bar{\mathbf{z}}\|_2^2 < \gamma^2 \|\mathbf{w}\|_2^2$ for all non-zero $\mathbf{w}(i, j) \in R^m$.

where $\|\bar{\mathbf{z}}\|_2^2 = \mathbb{E} \left\{ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} |\bar{z}_{i,j}|^2 \right\}$, $\|\mathbf{w}\|_2^2 = \mathbb{E} \left\{ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} |w_{i,j}|^2 \right\}$.

Lemma 1 Given a symmetric matrix $\mathbf{\Omega}$ and two matrices $\mathbf{\Psi}$ and $\mathbf{\Upsilon}$, consider the problem of finding some matrix \mathbf{G} such that

$$\mathbf{\Omega} + \mathbf{\Psi} \mathbf{G} \mathbf{\Upsilon} + (\mathbf{\Psi} \mathbf{G} \mathbf{\Upsilon})^T < 0$$

Then (4) is solvable for \mathbf{G} if and only if

$$\mathbf{\Psi}^\perp \mathbf{\Omega} \mathbf{\Psi}^{\perp T} < 0, \quad \mathbf{\Upsilon}^{T\perp} \mathbf{\Omega} \mathbf{\Upsilon}^{T\perp T} < 0 \quad (4)$$

Main results:

Theorem 3.1. Consider the error system (3) under Assumption 1, for a given a scalar $\gamma > 0$, the error system (3) is mean-square asymptotically stable with H_∞ performance, if there exist positive definite symmetric matrices $\mathbf{P}_m = \text{diag}\{\mathbf{P}_m^h, \mathbf{P}_m^v, \tilde{\mathbf{P}}_m^h, \tilde{\mathbf{P}}_m^v\} \in R^{(n+\tilde{n}) \times (n+\tilde{n})}$, $\mathbf{Q} = \text{diag}\{\mathbf{Q}^h, \mathbf{Q}^v, \tilde{\mathbf{Q}}^h, \tilde{\mathbf{Q}}^v\} \in R^{(n+\tilde{n}) \times (n+\tilde{n})}$, $\mathbf{P} = (\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_s)$, $m \in S$, such that the following linear matrix inequality holds for

$$\begin{bmatrix} -\bar{\mathbf{P}}_m & \bar{\mathbf{P}}_m \bar{\mathbf{A}}_m & \bar{\mathbf{P}}_m \bar{\mathbf{A}}_{d1m} & \bar{\mathbf{P}}_m \bar{\mathbf{A}}_{d2m} & \bar{\mathbf{P}}_m \bar{\mathbf{B}}_m & 0 \\ * & -\bar{\mathbf{P}}_m + \mathbf{Q} & 0 & 0 & 0 & \bar{\mathbf{C}}_m^T \\ * & * & -\bar{\mathbf{Q}}^h & 0 & 0 & \bar{\mathbf{C}}_{d1m}^T \\ * & * & * & -\bar{\mathbf{Q}}^v & 0 & \bar{\mathbf{C}}_{d2m}^T \\ * & * & * & * & -\gamma^2 \mathbf{I} & \bar{\mathbf{D}}_m^T \\ * & * & * & * & * & -\mathbf{I} \end{bmatrix} < 0 \quad (5)$$

where $\bar{\mathbf{P}}_m = \sum_{n=1}^s p_{mn} \mathbf{P}_n$, $\mathbf{P}_n = \text{diag}\{\mathbf{P}^h(r_{i+1,j}), \mathbf{P}^v(r_{i,j+1}), \tilde{\mathbf{P}}^h(r_{i+1,j}), \tilde{\mathbf{P}}^v(r_{i,j+1})\}$, $\bar{\mathbf{Q}}^h = \text{diag}\{\mathbf{Q}^h, \tilde{\mathbf{Q}}^h\}$,

$$\bar{Q}^v = \text{diag}\{Q^v, \tilde{Q}^v\}, \quad .$$

Proof : Consider system (3), let $w(t) = 0$ and the mode be m at time t , that is $r_{i,j} = m \in S$.

We can see that $\{(\bar{x}(i, j), r(t, j)), t \geq 0\}$ is not a Markov process with initial state $(X(0), R(0))$.

Now, we define a stochastic Lyapunov functional $V(\cdot)$ as follows:

Define $\Phi(t) = [\bar{x}^T(i, j) \quad \bar{x}_{d_1}^{h^T}(i, j) \quad \bar{x}_{d_2}^{v^T}(i, j)]^T$, by performing some simple arithmetic and Schur complement we can get:

$$\begin{aligned} \Delta V(\bar{x}(i, j), m) &= E\{V(\bar{x}'(i, j), n)\} - V(\bar{x}(i, j), m) \\ &= \sum_{n=1}^s p_{mn} \bar{x}^T(i, j) P_n \bar{x}'(i, j) - \bar{x}^T(i, j) P_m \bar{x}(i, j) + \sum_{\theta=1}^{d_1} \left[x^{h^T}(i+1-\theta, j) Q^h x^h(i+1-\theta, j) \right. \\ &\quad \left. - x^{h^T}(i-\theta, j) Q^h x^h(i-\theta, j) \right] + \sum_{\theta=1}^{d_2} \left[x^{v^T}(i, j+1-\theta) Q^v x^v(i, j+1-\theta) \right. \\ &\quad \left. - x^{v^T}(i, j-\theta) Q^v x^v(i, j-\theta) \right] + \sum_{\theta=1}^{d_1} \tilde{x}^{h^T}(i+1-\theta, j) \tilde{Q}^h \tilde{x}^h(i+1-\theta, j) \\ &\quad - \sum_{\theta=1}^{d_1} \tilde{x}^{h^T}(i+1-\theta, j) \tilde{Q}^h \tilde{x}^h(i-\theta, j) + \sum_{\theta=1}^{d_2} \tilde{x}^{v^T}(i, j+1-\theta) \tilde{Q}^v \tilde{x}^v(i, j+1-\theta) \\ &\quad - \sum_{\theta=1}^{d_2} \tilde{x}^{v^T}(i, j-\theta) \tilde{Q}^v \tilde{x}^v(i, j-\theta) \\ &= \bar{x}^T(i, j) \bar{P}_m \bar{x}'(i, j) - \bar{x}^T(i, j) (P_m + Q) \bar{x}(i, j) - \bar{x}_{d_1}^{h^T}(i, j) \bar{Q}^h \bar{x}_{d_1}^h(i, j) - \bar{x}_{d_2}^{v^T}(i, j) \bar{Q}^v \bar{x}_{d_2}^v(i, j) \\ &= \Phi^T(i, j) \Pi_m \Phi(i, j) \end{aligned}$$

$$\text{where, } \Pi_m = \begin{bmatrix} -P_m + Q + \bar{A}_m^T \bar{P}_m \bar{A}_m & \bar{A}_m^T \bar{P}_m \bar{A}_{d1m} & \bar{A}_m^T \bar{P}_m \bar{A}_{d2m} \\ * & -\bar{Q}^h + \bar{A}_{d1m}^T \bar{P}_m \bar{A}_{d1m} & \bar{A}_{d1m}^T \bar{P}_m \bar{A}_{d2m} \\ * & * & -\bar{Q}^v + \bar{A}_{d2m}^T \bar{P}_m \bar{A}_{d2m} \end{bmatrix}$$

Define $\Gamma(i, j) = [\bar{x}^T(i, j) \quad \bar{x}_{d_1}^{h^T}(i, j) \quad \bar{x}_{d_2}^{v^T}(i, j) \quad w^T(i, j)]^T$, puts the expression of $z(i, j)$ in J

Then

$$\begin{aligned} J &= E\{z^T(i, j) z(i, j) - \gamma^2 w(i, j)^T w(i, j) + V(\bar{x}'(i, j), n) - V(\bar{x}(i, j), m)\} \\ &= \Gamma^T(i, j) \bar{\Pi}_m \Gamma(i, j) \end{aligned}$$

where

$$\bar{\Pi}_m = \begin{bmatrix} -P_m + Q & 0 & 0 & 0 \\ * & -\bar{Q}^h & 0 & 0 \\ * & * & -\bar{Q}^v & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \bar{A}_m^T \\ \bar{A}_{d1m}^T \\ \bar{A}_{d2m}^T \\ \bar{B}_m^T \end{bmatrix} \bar{P}_m \begin{bmatrix} \bar{A}_m \\ \bar{A}_{d1m} \\ \bar{A}_{d2m} \\ \bar{B}_m \end{bmatrix}^T + \begin{bmatrix} \bar{C}_m^T \\ \bar{C}_{d1m}^T \\ \bar{C}_{d2m}^T \\ \bar{D}_m^T \end{bmatrix} \begin{bmatrix} \bar{C}_m \\ \bar{C}_{d1m} \\ \bar{C}_{d2m} \\ \bar{D}_m \end{bmatrix}^T < 0$$

Using Schur complement, the inequality (5) guarantees $\bar{\Pi}_m < 0$. Then we have $J < 0$, which

means for every $r(i, j) = m \in S$ we have

$$E\{V(\bar{x}'(i, j), n)\} < E\{V(\bar{x}(i, j), m) - z^T(i, j) z(i, j) + \gamma^2 w(i, j)^T w(i, j)\}$$

Considering the zero boundary condition, by using this relationship iteratively and performing superposition of the two sides of inequalities from $j=0$ to $j=k+1$, we can get

$$\begin{aligned} & \mathbb{E} \left\{ \sum_{j=0}^{k+1} \left[|\mathbf{x}_{k+1-j,j}^h|^2 + |\mathbf{x}_{k+1-j,j}^v|^2 + |\tilde{\mathbf{x}}_{k+1-j,j}^h|^2 + |\tilde{\mathbf{x}}_{k+1-j,j}^v|^2 \right] \right\} \\ & \leq \beta \sum_{j=0}^k \alpha^j \mathbb{E} \left[|\mathbf{x}_{0,k-j}^h|^2 + |\mathbf{x}_{k-j,0}^v|^2 + |\tilde{\mathbf{x}}_{0,k-j}^h|^2 + |\tilde{\mathbf{x}}_{k-j,0}^v|^2 + \sum_{\theta=1}^{d_1} \left[|\mathbf{x}_{-\theta,k-j}^h|^2 + |\tilde{\mathbf{x}}_{-\theta,k-j}^h|^2 \right] \right. \\ & \quad \left. + \sum_{\theta=1}^{d_2} \left[|\mathbf{x}_{k-j,-\theta}^v|^2 + |\tilde{\mathbf{x}}_{k-j,-\theta}^v|^2 \right] - \mathbf{z}^T(k-j,j) \mathbf{z}(k-j,j) + \gamma^2 \mathbf{w}(k-j,j)^T \mathbf{w}(k-j,j) \right] \\ & \quad + \sum_{j=0}^k \gamma^2 \mathbf{w}(k-j,j)^T \mathbf{w}(k-j,j) \end{aligned}$$

Summing up the two sides of the above inequalities from $k=0$ to $k=s$, we have

$$\begin{aligned} \mathbb{E} \left\{ \sum_{k=0}^s \sum_{j=0}^k \mathbf{z}^T(k-j,j) \mathbf{z}(k-j,j) \right\} & < \mathbb{E} \left\{ \sum_{k=0}^s \sum_{j=0}^k \gamma^2 \mathbf{w}(k-j,j)^T \mathbf{w}(k-j,j) \right\} \\ & - \mathbb{E} \left\{ \sum_{j=0}^{s+1} \left[\mathbf{x}_{s+1-j,j}^T \mathbf{P}(r_{s+1-j,j}) \mathbf{x}_{s+1-j,j} \right] \right\} \end{aligned}$$

Therefore, when $s \rightarrow \infty$, we have

$$\mathbb{E} \left\{ \sum_{k=0}^{\infty} \sum_{j=0}^k \mathbf{z}^T(k-j,j) \mathbf{z}(k-j,j) \right\} < \mathbb{E} \left\{ \sum_{k=0}^{\infty} \sum_{j=0}^k \gamma^2 \mathbf{w}(k-j,j)^T \mathbf{w}(k-j,j) \right\}$$

that is, $\|\mathbf{z}\|_2^2 < \gamma^2 \|\mathbf{w}\|_2^2$ for all non-zero $\mathbf{w} = \{w_{ij}\} \in L_2\{[0, \infty), [0, \infty)\}$, and the proof is concluded.

Theorem Consider the mean-square asymptotically stable 2D jump system (1). Given a constant $\gamma > 0$, there exists a reduced \tilde{n} th-order system (2) solves the H_∞ model reduction problem if there exists a block-diagonal matrix $\mathbf{X} = (\bar{\mathbf{X}}_1, \bar{\mathbf{X}}_2, \dots, \bar{\mathbf{X}}_s)$ with $\bar{\mathbf{X}}_m > 0$,

$\mathbf{P} = (\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_s)$ with $\mathbf{P}_m > 0$, $\mathbf{P}_m = \text{diag}\{\mathbf{P}_m^h, \mathbf{P}_m^v, \tilde{\mathbf{P}}_m^h, \tilde{\mathbf{P}}_m^v\}$, $\mathbf{Q} = \text{diag}\{\mathbf{Q}^h, \mathbf{Q}^v, \tilde{\mathbf{Q}}^h, \tilde{\mathbf{Q}}^v\}$,

$\forall m \in S$, such that the following linear matrix inequality holds for

$$\begin{bmatrix} -E\bar{\mathbf{X}}_m E^T & E\mathbf{A}_{0m} & E\mathbf{A}_{d10m} & E\mathbf{A}_{d20m} & E\mathbf{B}_{0m} \\ * & -\mathbf{P}_m + \mathbf{Q} & 0 & 0 & 0 \\ * & * & -\bar{\mathbf{Q}}^h & 0 & 0 \\ * & * & * & -\bar{\mathbf{Q}}^v & 0 \\ * & * & * & * & -\gamma^2 \mathbf{I} \end{bmatrix} < 0 \quad (7)$$

$$\begin{bmatrix} -\bar{X}_m & A_{0m} & A_{d10m} & A_{d20m} & 0 \\ * & -P_m + Q & 0 & 0 & C_{0m}^T \\ * & * & -\bar{Q}^h & 0 & C_{d10m}^T \\ * & * & * & -\bar{Q}^v & C_{d20m}^T \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (8)$$

$$\bar{X}_m \sum_{n=1}^s p_{mn} P_n = I \quad (9)$$

Furthermore, if matrix \bar{X}_m , P_m , Q are the solutions of (7)-(9), then a reduced order model can be written as

$$\begin{aligned} G_m &= -R_m^{-1} \Psi_m^T A_m \Upsilon_m^T (\Upsilon_m A_m \Upsilon_m^T)^{-1} + R_m^{-1} V_m^{1/2} L_m (\Upsilon_m A_m \Upsilon_m^T)^{-1/2} \\ A_m &= (\Psi_m R_m^{-1} \Psi_m^T - Q_m)^{-1} \\ V_m &= R_m - \Psi_m^T (A_m - A_m \Upsilon_m^T (\Upsilon_m A_m \Upsilon_m^T)^{-1} \Upsilon_m A_m) \Psi_m \end{aligned} \quad (10)$$

where $R_m > 0$ such that $A_m > 0$ and L_m is any matrix satisfying $\|L_m\| < 1$, and

$$\begin{aligned} A_{0m} &= \begin{bmatrix} A_m & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{d10m} = \begin{bmatrix} A_{d1m} & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{d20m} = \begin{bmatrix} A_{d2m} & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{0m} = \begin{bmatrix} B_m \\ 0 \end{bmatrix}, \\ G_m &= \begin{bmatrix} \tilde{D}_m & \tilde{C}_m & \tilde{C}_{d1m} & \tilde{C}_{d2m} \\ \tilde{B}_m & \tilde{A}_m & \tilde{A}_{d1m} & \tilde{A}_{d2m} \end{bmatrix}, \quad C_{0m} = \begin{bmatrix} C_m^T \\ 0 \end{bmatrix}, \quad C_{d10m} = \begin{bmatrix} C_{d1m}^T \\ 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0_{n,l} & 0_{n,\tilde{n}} \\ 0_{\tilde{n},l} & I_{\tilde{n}} \end{bmatrix}, \\ C_{d20m} &= \begin{bmatrix} C_{d2m} & 0 \end{bmatrix}, \quad D_{0m} = D_m, \quad \Upsilon_m = \begin{bmatrix} 0 & H & M & W & N & 0 \end{bmatrix}, \quad S = \begin{bmatrix} -I_l & 0_{l,\tilde{n}} \end{bmatrix}, \end{aligned}$$

$$\Omega_m = \begin{bmatrix} -\bar{P}_m & \bar{P}_m A_{0m} & \bar{P}_m A_{d10m} & \bar{P}_m A_{d20m} & \bar{P}_m B_{0m} & 0 \\ * & -P_m + Q & 0 & 0 & 0 & C_{0m}^T \\ * & * & -\bar{Q}^h & 0 & 0 & C_{d10m}^T \\ * & * & * & -\bar{Q}^v & 0 & C_{d20m}^T \\ * & * & * & * & -\gamma^2 I & D_{0m}^T \\ * & * & * & * & * & -I \end{bmatrix}, \quad \Psi_m = \begin{bmatrix} \bar{P}_m F \\ 0 \\ 0 \\ 0 \\ 0 \\ S \end{bmatrix},$$

$$N = \begin{bmatrix} I_m \\ 0_{\tilde{n},m} \\ 0_{\tilde{n}_1,m} \\ 0_{\tilde{n}_2,m} \end{bmatrix}, \quad H = \begin{bmatrix} 0_{m,n} & 0_{m,\tilde{n}} \\ 0_{\tilde{n},n} & I_{\tilde{n}} \\ 0_{\tilde{n}_1,n} & 0_{\tilde{n}_1,\tilde{n}} \\ 0_{\tilde{n}_2,n} & 0_{\tilde{n}_2,\tilde{n}} \end{bmatrix}, \quad M = \begin{bmatrix} 0_{m,n_1} & 0_{m,\tilde{n}_1} \\ 0_{\tilde{n},n_1} & 0_{\tilde{n},\tilde{n}_1} \\ 0_{\tilde{n}_1,n_1} & I_{\tilde{n}_1} \\ 0_{\tilde{n}_2,n_1} & 0_{\tilde{n}_2,\tilde{n}_1} \end{bmatrix}, \quad W = \begin{bmatrix} 0_{m,n_2} & 0_{m,\tilde{n}_2} \\ 0_{\tilde{n},n_2} & 0_{\tilde{n},\tilde{n}_2} \\ 0_{\tilde{n}_1,n_2} & 0_{\tilde{n}_1,\tilde{n}_2} \\ 0_{\tilde{n}_2,n_2} & I_{\tilde{n}_2} \end{bmatrix} \quad (11)$$

Proof: From Theorem 3.1, we know that there exists a reduced \tilde{n} th-order system (2) such that the error system (3) has H_∞ performance if there exist positive definite symmetric matrices P_m , \bar{Q}^h ,

\bar{Q}^v , $m \in S$ such that (5) holds. It can be seen from (11) that

$$\begin{bmatrix} \bar{A}_m \\ \bar{A}_{d1m} \\ \bar{A}_{d2m} \\ \bar{B}_m \end{bmatrix} = \begin{bmatrix} A_{0m} \\ A_{d10m} \\ A_{d20m} \\ B_{0m} \end{bmatrix} + FG_m \begin{bmatrix} H \\ M \\ W \\ N \end{bmatrix}, \quad \begin{bmatrix} \bar{C}_m \\ \bar{C}_{d1m} \\ \bar{C}_{d2m} \\ \bar{D}_m \end{bmatrix} = \begin{bmatrix} C_{0m} \\ C_{d10m} \\ C_{d20m} \\ D_{0m} \end{bmatrix} + SG_m \begin{bmatrix} H \\ M \\ W \\ N \end{bmatrix} \quad (12)$$

Furthermore, noticing that (12), (5) can be rewritten as

$$\Omega_m + \Psi_m G_m \Upsilon_m + (\Psi_m G_m \Upsilon_m)^T < 0 \quad (13)$$

Then from Lemma 1, (13) is equivalent to

$$\Psi_m^\perp \Omega_m \Psi_m^{\perp T} < 0, \quad \Upsilon_m^{T\perp} \Omega_m \Upsilon_m^{T\perp} < 0$$

Let $\bar{X}_m = \bar{P}_m^{-1}$, it can be seen that (7) is equivalent to $\Psi_m^\perp \Omega_m \Psi_m^{\perp T} < 0$ and (8) is equivalent to

$$\Upsilon_m^{T\perp} \Omega_m \Upsilon_m^{T\perp} < 0. \text{ Hence, the results can be obtained by Lemma 1.}$$

It should be noted that the obtained conditions in Theorem 2 are not LMI conditions due to the equations in (9). However, with the result of a cone complementarily linearization algorithm, we can solve this feasibility problem by formulating it into a linear optimization problem subject to LMI constraints.

Problem 1:
$$\min \sum_{m=1}^s \sum_{n=1}^s p_{mn} \text{Trace}(\bar{X}_m P_n) \quad (14)$$

s.t. i (7)-(8)

ii
$$\begin{bmatrix} \sum_{n=1}^s p_{mn} P_n & I \\ I & \bar{X}_m \end{bmatrix} \geq 0$$

Since the employ the method of alternating projections to solve the 2D jump system H_∞ model reduction problem can not guarantee global convergence, in this paper, we can use the following Algorithm to solve the above nonlinear problem.

Model reduction algorithm

For $X = (\bar{X}_1, \bar{X}_2, \dots, \bar{X}_s)$ with $\bar{X}_m > 0$ and $P = (P_1, P_2, \dots, P_s)$ with $P_m > 0$, $m \in S$, define a convex set by a set of LMIs as

$$\xi_{(X,P)}^d = \{(X, P): \text{LMI(8), LMI(9), } \bar{X}_m > 0, P_m > 0, \text{ for all } m \in S \}$$

It can be seen from Theorem 2, the H_∞ reduced order models for 2D jump linear systems (1) can be obtained if there exist $X = (\bar{X}_1, \bar{X}_2, \dots, \bar{X}_s)$ and $P = (P_1, P_2, \dots, P_s)$ such that

$$(X, P) \in \xi_{(X,P)}^d, \quad \bar{X}_m \sum_{n=1}^s p_{mn} P_n = I, \text{ for all } m \in S \} \quad (15)$$

is feasible.

With the above expressions, the following algorithm is proposed to solve the 2D jump system H_∞ model reduction problem:

Step1: Choose the initial values for the matrix pair (P_0, X_0) , the order of the reduced-order \tilde{n} and the H_∞ norm bound γ ;

Step2: Define the linear function $f_k(X, P) = \sum_{m=1}^s \sum_{n=1}^s p_{mn} \text{Trace}(\bar{X}_{mk} P_n + P_{nk} \bar{X}_m)$

Step3: Find (X_{k+1}, P_{k+1}) solving the following convex programming: $\min_{(X, P) \in \mathcal{E}_s^d(X, P_n)} \{f_k(X, P)\}$

Step4: If f_k converges, then exit; otherwise, set $k = k + 1$; and go to step 2.

Step5: Construct a reduced-order model based on (10).

It can be seen that step 1 is a simple LMI problem, and step 3 is a convex programming with LMI constraints. From the explanation in [7], f_k is decreasing and bounded below by $2s(n + \tilde{n})$. Once it converges, then (14) is feasible, which implies that the H_∞ model reduction problem is solvable for a given $\gamma > 0$.

Numerical example

In this section, we present a numerical example to illustrate the effectiveness of the proposed method. It is assumed that the system has two operation modes.

For mode 1, the system matrices are given by:

$$A_1 = \begin{bmatrix} 0.5 & 0.01 & 0.01 & 0 \\ 0 & 0.6 & 0 & 0.01 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.4 \end{bmatrix}, \quad A_{d11} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{d21} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \\ 0.02 & 0 \\ 0 & 0.03 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1.2 & 0.4 & 0.6 & 0.9 \\ 0.4 & 0.5 & 0.6 & 0.1 \end{bmatrix}, \quad C_{d11} = \begin{bmatrix} 0.12 & 0.04 \\ 0.04 & 0.05 \end{bmatrix}, \quad C_{d21} = \begin{bmatrix} 0.06 & 0.09 \\ 0.06 & 0.01 \end{bmatrix},$$

$$B_1 = [0.1 \quad 0.7 \quad 1.3 \quad 0.5]^T, \quad D_1 = [0.01 \quad 0.02]^T$$

For mode 2, the system matrices are given by:

$$A_2 = \begin{bmatrix} -0.3 & 0.01 & 0 & 0 \\ 0 & -0.7 & 0 & 0.02 \\ 0.03 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & -0.4 \end{bmatrix}, \quad A_{d12} = \begin{bmatrix} 0.01 & 0.01 \\ 0 & -0.02 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{d22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.02 & 0 \\ 0 & -0.01 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1.1 & 0.5 & 0.7 & 1.9 \\ 0.1 & 0.3 & 0.4 & 0.4 \end{bmatrix},$$

$$C_{d12} = \begin{bmatrix} 0.012 & 0.004 \\ 0.004 & 0.005 \end{bmatrix}, \quad C_{d22} = \begin{bmatrix} 0.006 & 0.009 \\ 0.006 & 0.001 \end{bmatrix}, \quad B_2 = [0.3 \quad 0.7 \quad 1.2 \quad 0.1]^T, \quad D_2 = [0.2 \quad 0.5]^T$$

The modal transfer matrix is given by: $\Pi = \begin{bmatrix} 0.1 & 0.9 \\ 0.55 & 0.45 \end{bmatrix}$

Our purpose is to find a (2h,1v) reduced order model. For given $\gamma = 5.8943$, using the CCL

algorithm, we solve the matrix inequalities (7-9), for $m=1, 2$, let $L_1 = L_2 = [L_{11} \quad L_{12}]$, where,

$L_{11} = I_5$, $L_{12} = [I_2 \quad I_2 \quad [0 \quad 1]^T]^T$, we can obtain the solutions as

$$R_1 = \begin{bmatrix} 0.1057 & -0.1216 & 0.0002 & 0.0000 & 0.0009 \\ -0.1216 & 0.2952 & 0.0001 & -0.0003 & 0.0026 \\ 0.0002 & 0.0001 & 0.4247 & -0.0000 & -0.0023 \\ 0.0000 & -0.0003 & -0.0000 & 0.4247 & -0.0000 \\ 0.0009 & 0.0026 & -0.0023 & -0.0000 & 0.4250 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 7.7719 & 0.0625 & -0.0001 & 0.0010 & -0.0000 \\ 0.0625 & 7.5170 & 0.0008 & -0.0001 & 0.0013 \\ -0.0001 & 0.0008 & 5.5630 & 0.0002 & -0.1084 \\ 0.0010 & -0.0001 & 0.0002 & 5.5634 & 0.0014 \\ -0.0000 & 0.0013 & -0.1084 & 0.0014 & 5.5616 \end{bmatrix}$$

Then the (2h,1v) reduced order model can be computed from Theorem 3.2, which is given by

$$\tilde{A}_1 = \begin{bmatrix} 0.5186 & -0.0285 & -0.0031 \\ 0.0046 & 0.1051 & 0.0041 \\ -0.0016 & 0.0001 & 0.1351 \end{bmatrix}, \quad \tilde{A}_2 = \begin{bmatrix} -0.2509 & -0.0249 & 0.0003 \\ 0.7945 & 0.1477 & 0.0045 \\ -0.0487 & 0.0005 & 0.1659 \end{bmatrix}, \quad \tilde{A}_{d11} = \begin{bmatrix} 0.0060 & 0.0002 \\ -0.0128 & 0.0005 \\ 0.0002 & -0.0003 \end{bmatrix},$$

$$\tilde{A}_{d21} = \begin{bmatrix} 0.0626 \\ -0.0299 \\ 0.1335 \end{bmatrix}, \quad \tilde{A}_{d22} = \begin{bmatrix} -0.0013 & 0.0005 \\ -0.0045 & -0.0028 \\ 0.0001 & -0.0016 \end{bmatrix}, \quad \tilde{A}_{d23} = \begin{bmatrix} 0.0688 \\ -0.0302 \\ 0.2063 \end{bmatrix}, \quad \tilde{B}_1 = \begin{bmatrix} -0.0288 \\ 0.1075 \\ -0.004 \end{bmatrix}, \quad \tilde{B}_2 = \begin{bmatrix} -0.0306 \\ 0.1820 \\ -0.0010 \end{bmatrix},$$

$$\tilde{C}_{d11} = \begin{bmatrix} 0.1342 & -0.0000 \\ 0.0004 & 0.1336 \end{bmatrix}, \quad \tilde{C}_1 = \begin{bmatrix} 0.0079 & 0.0031 & 0.0004 \\ -0.0035 & 0.0004 & -0.0005 \end{bmatrix}, \quad \tilde{C}_{d12} = \begin{bmatrix} 0.0000 \\ -0.0002 \end{bmatrix}, \quad \tilde{C}_{d22} = \begin{bmatrix} -0.0001 \\ -0.0012 \end{bmatrix},$$

$$\tilde{C}_2 = \begin{bmatrix} 0.0391 & 0.0009 & -0.0001 \\ -0.0810 & -0.0016 & -0.0012 \end{bmatrix}, \quad \tilde{C}_{d12} = \begin{bmatrix} 0.1662 & 0.0001 \\ -0.0008 & 0.2061 \end{bmatrix}, \quad \tilde{D}_2 = \begin{bmatrix} 0.2064 \\ 0.2038 \end{bmatrix}, \quad \tilde{D}_1 = \begin{bmatrix} 0.1333 \\ 0.1338 \end{bmatrix}$$

It is assumed the disturbance input $w(t)$ is expressed as $w(i, j) = e^{-0.1(i+j)}$. Define the initial conditions are $x(0) = [0 \quad 0 \quad 0]^T$ and $r(0) = 1$. Simulation results are shown in the following figures.

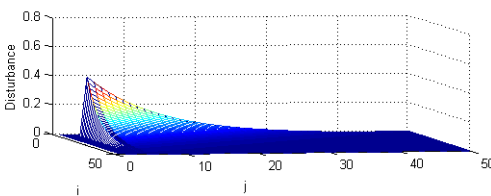
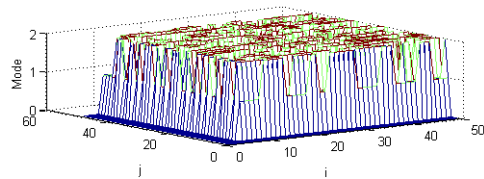


FIG. 1: Trajectories of $r(t)$ and $w(t)$

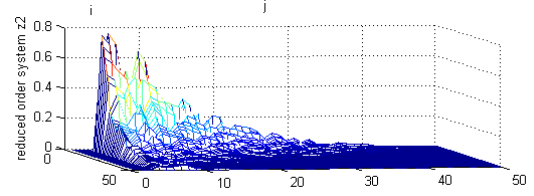
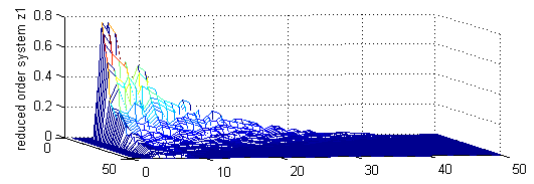


FIG. 3: Trajectories of system output of reduced system

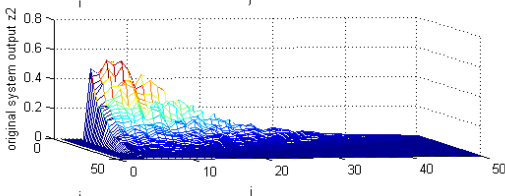
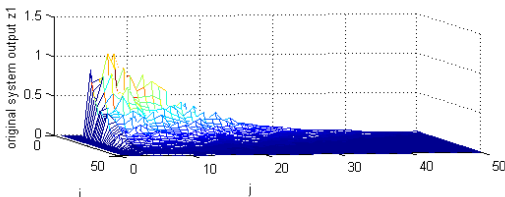


FIG. 2: Trajectories of system output of original system

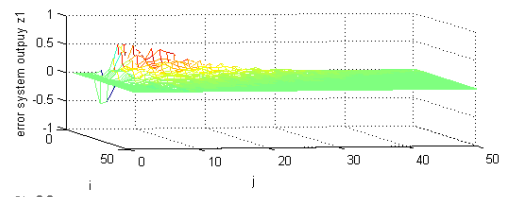


FIG. 4: Error output response of $\bar{z}(t)$

Analyzing from Figure 1, we can conclude that the conversion between the two modes is randomly and the disturbance input $w(t)$ is energy-bounded. Judging from the simulated curve of the Figure 2, it can be seen that the error filter system is asymptotic stable for nondeterminacy, and the peak gain of the error $\bar{z}(t)$ is no more than the sub-optimum value $\gamma = 1.3938$. The simulation results imply that the desired goal is well achieved.

Conclusions

This paper extends the results obtained for one-dimensional Markovian jump systems to investigate the problem of H_∞ model reduction for a class for linear discrete time 2D Markovian jump systems with state delays in Roesser model which are time-varying and mode-independent. The jump parameters are modeled by a finite-state Markov process. A reduced-order model with the same randomly jumping parameters is proposed which can make the error systems stochastically stable with a prescribed H_∞ performance. Then a sufficient condition in terms of linear matrix inequalities (LMIs) plus matrix inverse constraints are derived for the existence of a solution to the reduced-order model problems. Since these obtained conditions are not expressed as strict LMIs, the cone complementarity linearization (CCL) method is exploited to cast them into nonlinear minimization problems subject to LMI constraints, which can be readily solved by standard numerical soft ware. A numerical example is given to illustrate the design procedures.

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