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Nonlinear Uncertain Systems Control Research Based on a Novel SMC Method

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Abstract. An improved sliding mode controller is designed for a class of nonlinear uncertain systems and the sliding mode is proved existing and accessible by theoretical analysis. In addition, a method to improve the control precision is proposed after analyzing the stability. The method can not only weaken the chattering, but also reduce the difficulty and workload of designing the controller. Finally, the simulation results show that the method is correct and effective, and then the robustness of the system can be improved.

Introduction

In actual control, many systems have strong nonlinear and model objects often have uncertainties, Sliding Mode Control (SMC) just provides effective solutions to these problems, SMC has been widely used as it has strong robustness to the system with parameter uncertainties and external disturbances. Such as in the literature [2], SMC was applied to uncertain systems to realize robust control. However, the chattering problem is a big obstacle in the practical application of SMC. In order to weaken the chattering phenomenon of SMC, the effective methods were presented by many experts and scholars in China and abroad. In the literature [3], the author puts forward the concept of boundary layer for a class of nonlinear systems, namely, the normal SMC is used outside the boundary layer and the continuous state feedback control is used within the boundary layer, which can well weaken the chattering, but at the same time will produce certain steady-state error. To this, literature [4] deduces the scope of the steady-state error based on the literature [5,6],and design an improved integral type sliding mode surface on the basis of literature [7,8].The effect of this method is good when the disturbance is ordinary step signal, but when the disturbance is nonlinear signal, the accuracy of the system response is not very ideal.

In this paper, an improved sliding surface is proposed by introducing the slope of the sliding surface. The relationship between the steady-state error of n-order nonlinear uncertain system and the slope of sliding surface and the boundary layer thickness was deduced and discussed. On the basis, a sliding mode controller is designed. Theoretical analysis and the simulation curves show that adopting the improved sliding surface introducing the slope of the sliding surface and the controller this paper designed not only the system is controlled effectively but also the chattering is eliminated. Comparing with the literature [4],this paper can improve the adjusting time, reduce the error and restrain the chattering more effectively, which shows the powerful robustness.

System Description

Consider the following n-order nonlinear uncertain system:

$$\begin{cases} x^{(n)}(t) = f(X(t)) + g(X(t))u(t) + d(X(t)) \\ y(t) = x(t) \end{cases} \tag{1}$$

where $X(t) = \left[x(t), \dot{x}(t), \cdots, x^{(n-1)}(t) \right]^T \in R^n$ is the state of the system, $x(t) \in R$; $u(t) \in R^m$ is the control, $g(X(t))$ is the control gain, invertible; $f(X(t))$ is the nonlinear uncertain bounded function, $d(X(t))$ is the unknown bounded disturbance.

Let

$$f(X(t)) = \tilde{f}(X(t)) + \Delta f(X(t)) \tag{2}$$

Where $\tilde{f}(X(t))$ is the estimate function of $f(X(t))$, $\Delta f(X(t))$ is the uncertain item of the system, then we have

$$\left|\Delta f(X(t))\right| \le F(X) \tag{3}$$

$$\left|d(X(t))\right| \le D(X) \tag{4}$$

Where $F(X)$ and $D(X)$ are the upper bound functions of $\Delta f(X(t))$ and $d(X(t))$, respectively. We want to show in this paper that the output of the system $y(t)$ can track the reference input signal $x_r(t)$,and the system can have good steady performance in the presence of nonlinear uncertain $f(X(t))$ and external disturbance. Here, $x_r(t) \in R$ is continuous differentiable.

Define the tracking error $e_1(t) = y(t) - x_r(t) = x(t) - x_r(t)$,then we have the error vector is

$$\begin{aligned} e^T(t) &= \left[e_1(t), e_2(t), \cdots, e_n(t)\right] = \left[e_1(t), \dot{e}_1(t), \cdots, e_1^{(n-1)}(t) \right] \\ &= \left[x(t) - x_r(t), \dot{x}(t) - \dot{x}_r(t), \cdots, x^{(n-1)}(t) - x_r^{(n-1)}(t) \right] \in R^n \end{aligned} \tag{5}$$

The Sliding Mode Controller Design

To reduce the steady-state error, the slope of the sliding surface λ is introduced to the traditional sliding mode surface and form the sliding surface below

$$S(t) = \lambda \sum_{i=1}^{n-1} c_i e_i + e_n \tag{6}$$

Where $\lambda > 0$ is the slope of the sliding surface, constant $c_1, c_2, \cdots, c_{n-1}$ are selected to make $p^{n-1} + c_{n-1}p^{n-2} + \cdots + c_2 p + c_1$ Hurwitz stability, p is Laplace operator.

Theorem 1. Consider the sliding surface given by (6) of the system described by (1),if the control signal is

$$u(t) = g^{-1}(X(t))\left[-\lambda\sum_{i=1}^{n-1} c_i e_{i+1} - \tilde{f}(X(t)) + x_r^{(n)}(t) - M(X)\cdot \mathrm{sgn}(S(t))\right] \quad (7)$$

Then sliding mode exists and it is accessible. Where $M(X) = F(X) + D(X) + \varepsilon$, $\varepsilon > 0$.

Proof: Taking the time derivative of (6) to obtain

$$\dot{S}(t) = \lambda\sum_{i=1}^{n-1} c_i e_{i+1} + \dot{e}_n = \lambda\sum_{i=1}^{n-1} c_i e_{i+1} + x^{(n)}(t) - x_r^{(n)}(t)$$

$$= \lambda\sum_{i=1}^{n-1} c_i e_{i+1} + f(X(t)) + g(X(t))u(t) + d(X(t)) - x_r^{(n)}(t) \quad (8)$$

Let $\dot{S}(t) = 0$,we obtain an equivalent controller $u_{eq}(t)$,that is

$$u_{eq}(t) = g^{-1}(X(t))\left[-\lambda\sum_{i=1}^{n-1} c_i e_{i+1} - f(X(t)) - d(X(t)) + x_r^{(n)}(t)\right]$$

$$= g^{-1}(X(t))\left[-\lambda\sum_{i=1}^{n-1} c_i e_{i+1} - \tilde{f}(X(t)) - \Delta f(X(t)) - d(X(t)) + x_r^{(n)}(t)\right] \quad (9)$$

Control signal such as (7) is given based on (9),then we have

$$S(t)\cdot\dot{S}(t) = S(t)\left[\Delta f(X(t)) + d(X(t)) - M(X)\cdot \mathrm{sgn}(S(t))\right]$$

$$= S(t)\left[\Delta f(X(t)) + d(X(t))\right] - M(X)\left|S(t)\right|$$

$$\le \left|S(t)\right|\cdot\left|\Delta f(X(t)) + d(X(t))\right| - \left[F(X) + D(X) + \varepsilon\right]\cdot\left|S(t)\right| < -\varepsilon\left|S(t)\right| < 0$$

Therefore, theorem 1 has been proved.

Saturation function is adopted to instead of sign function to eliminate the chattering,the saturation function is defined as

$$sat\left[S(t)/\mu\right] = \begin{cases} S(t)/\mu, \left|S(t)\right| \le \mu \\ \mathrm{sgn}(S(t)), \left|S(t)\right| > \mu \end{cases} \quad (10)$$

Then

$$u(t) = g^{-1}(X(t))\left[-\lambda\sum_{i=1}^{n-1} c_i e_{i+1} - \tilde{f}(X(t)) + x_r^{(n)}(t) - M(X)\cdot sat(S(t)/\mu)\right] \quad (11)$$

where μ is the boundary layer thickness.

The situation of the steady-state error when the system state converge to the internal boundary layer with the center of the sliding mode surface will be discussed in the following.

The Steady-state Error Analysis

Theorem 2. Consider the sliding surface given by (6) of the system described by (1),when $W(X(t))$ is constant or constant in the end, namely, when $\lim_{t\to\infty} W(X(t)) = l$ (l is constant),then we have

$$\lim_{t\to\infty} e(t) = l/(c_1\lambda\gamma) \tag{12}$$

Where, $W(X(t)) = \Delta f(X(t)) + d(X(t))$, $\gamma = M(X)/\mu$.

Proof: The track of system (1) within the boundary layer $|S(t)| \le \mu$

$$\dot{S}(t) = \lambda\sum_{i=1}^{n-1} c_i e_{i+1} + \dot{e}_n = \Delta f(X(t)) + d(X(t)) - M(X)\cdot[S(t)/\mu]$$

$$= W(X(t)) - \gamma \cdot S(\tag{13}$$

Take Laplace transform of (13), we obtain

$$S(s) = W(s)/(s+\gamma) \tag{14}$$

In the literature [4],we know when $W(X(t))$ is constant or constant in the end, $\lim_{t\to\infty} W(X(t)) = l$ (l is constant),then we have

$$\lim_{t\to\infty} S(t) = \lim_{s\to 0} s\cdot S(s) = l/\gamma \tag{15}$$

And according to $e_1(t) = x(t) - x_r(t)$ defined before, as the steady-state error is required, the influence of the initial state of the system can be neglected,(6) is expand into

$$S(t) = \lambda[c_1e_1(t) + c_2e_2(t) + \cdots + c_{n-1}e_{n-1}(t)] + e_n(t)$$

$$= \lambda[c_1e_1(t) + c_2\dot{e}_1(t) + \cdots + c_{n-1}e_1^{(n-2)}(t)] + e_1^{(n-1)}(t) \tag{16}$$

Take Laplace transform of (16),we obtain

$$S(s) = \lambda(c_1 + c_2s + \cdots + c_{n-1}s^{n-2} + s^{n-1})e_1(s)$$

Therefore, $e_1(s) = S(s)/[\lambda(c_1 + c_2s + \cdots + c_{n-1}s^{n-2} + s^{n-1})]$

According to the final value theorem, we get

$$\lim_{t\to\infty} e_1(t) = \lim_{s\to 0} s\cdot e_1(s) = \lim_{s\to 0} 1\Big/\left[\lambda(c_1 + c_2 s + \cdots + c_{n-1}s^{n-2} + s^{n-1})\right]\cdot(l/\gamma) = l/(c_1\lambda\gamma)$$

Therefore, theorem 2 has been proved.

In the following we analyze it, through $l/(c_1\lambda\gamma) = \mu\cdot l\big/\left[c_1\lambda M(X)\right]$,we know that the steady-state error of the system is inversely proportional to slope of the sliding surface λ and proportional to the boundary layer thickness μ .The smaller μ is, the smaller the steady-state error is so that the control effect is better, however, the chattering is increased; If λ is very large, which will weaken the advantage of powerful robustness of the sliding mode control, thus it can be seen that adjusting the value of λ and μ can enhance the control effect but the adjustment process is troublesome, so according to the analysis and derivation before, we can achieve an good control effect by an easy method, namely, after the value of λ and μ is initially identified, the constant coefficient c_1 can be adjusted, which can reduce the steady-state error further, enhance the robustness of the system and improve its steady-state performance.

Simulation Examples

Consider the n-order system described in (1), take the following 2-order system in which n=2 as an example:

$$\begin{cases} \ddot{x}(t) = \tilde{f}(X(t)) + \Delta f(X(t)) + g(X(t))u(t) + d(X(t)) \\ y(t) = x(t) \end{cases}$$

Where assume that $\tilde{f}(X(t)) = x_2$, $\Delta f(X(t)) = \cos(t)$, $d(X(t)) = 2\sin(t)$, tracking reference input $x_r(t) = \sin(t)$.

The improved sliding surface given by (6) and the designed controller given by (7) is adopted, let $\varepsilon = 10$, $F(X) = 1$, $D(X) = 10$, therefore, $M(X) = 21$,according to the above analysis, the following simulation of several cases.

Case 1: when the value of μ is large such as when $\mu = 0.2$,the simulation result is shown in Fig.1,which the error is large and the chattering exists.

Case 2:when the value of μ is small such as when $\mu = 0.05$,the simulation result is shown in Fig.2,which the error is small but the chattering increases.

Case 3:when $\mu = 0.02$, $\lambda = 3$, $c_1 = 5$,the simulation result is shown in Fig.3, the error is reduced further and the chattering is weakened at the same time.

Case 4:when $\mu = 0.02$, $\lambda = 1$, $c_1 = 10$,the simulation result is shown in Fig.4,not only the error is reduced, but also the chattering is eliminated.

Fig.5 is the curve that the output tracks the reference input signal in this case. "--" is the reference input signal, "-" is the output signal.

Through the five figures, it can be seen that they completely accords with the steady-state error analysis conclusion that before describes. Synthesizing the above situations and comparing with the literature [4],the response speed is improved, the adjusting time and steady-state error are reduced and the chattering is restrained more effectively, which shows the powerful robustness.

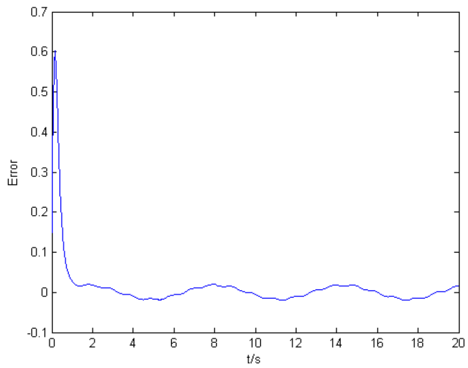

Fig.1 The error curve in case 1

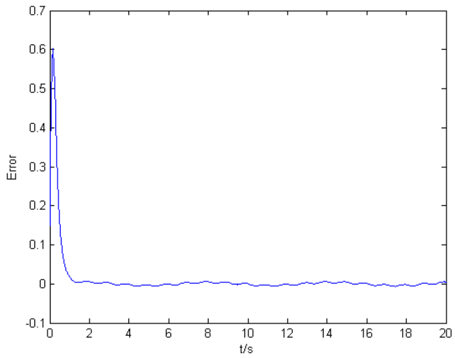

Fig.2 The error curve in case 2

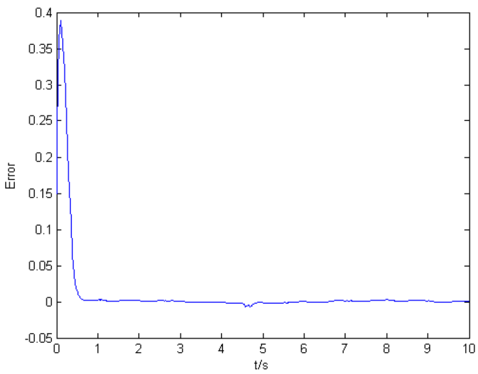

Fig.3 The error curve in case 3

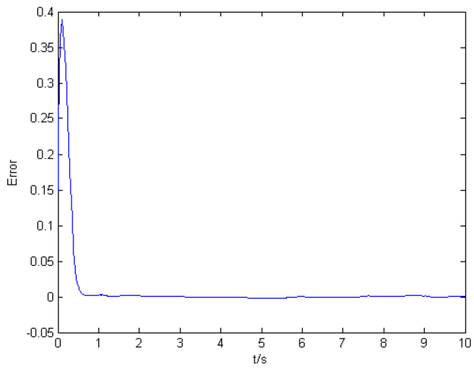

Fig.4 The error curve in case 4

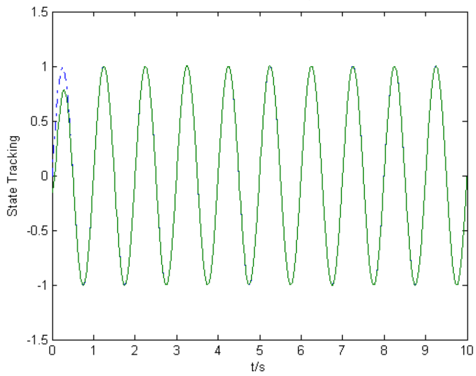

Fig.5 The output tracking error curve in case 4

6 Conclusion

In this paper, an improved sliding surface is proposed by introducing the slope of the sliding surface λ.On the basis; a sliding mode controller is designed. The above proposed method is feasible by theoretical analysis. Then, the steady-state error is discussed for a class of n-order nonlinear uncertain systems. The slope of sliding surface λ and the boundary layer thickness μ have bind effects mutually, so they are not easy to adjust. The analysis and simulation show that the method in this paper can adjust appropriately constant coefficient c_1 to realize the effective control of the system under the situation of the certain value of λ and μ .Comparing with the literature [4],this paper can improve the response speed and adjusting time, reduce the error further and restrain the chattering more effectively, which shows the powerful robustness and at the same time reduces the workload of adjusting value of λ and μ ,which is easy for the project realization.

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